Advanced Computation Models for Rule Based Networks

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Presentation Outline

Introduction
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Formal Models for Rule Based Networks
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Introduction

Modelling Aspects of Systemic Complexity

- non-linearity (input-output functional relationships)
- uncertainty (incomplete and imprecise data)
- dimensionality (large number of inputs and outputs)
- structure (interacting subsystems)

Complexity Management by Rule Based Systems

- model feasibility (achievable in the case of non-linearity)
- model accuracy (achievable in the case of uncertainty)
- model efficiency (problematic in the case of dimensionality)
- model transparency (problematic in the case of structure)

Types of Rule Based Systems

<u>Logical Connections</u>

- disjunctive antecedents and conjunctive rules
- conjunctive antecedents and conjunctive rules
- disjunctive antecedents and disjunctive rules
- conjunctive antecedents and disjunctive rules

<u>Inputs and Outputs</u>

- single-input-single-output
- single-input-multiple-output
- multiple-input-single-output
- multiple-input-multiple-output

Types of Rule Based Systems

Rule Base Properties

- completeness
- consistency
- exhaustiveness
- monotonicity

Rule Base Type

- single rule base (standard rule based system)
- chained rule bases (hierarchical rule based system)
- modular rule bases (rule based network)

Types of Rule Based Systems

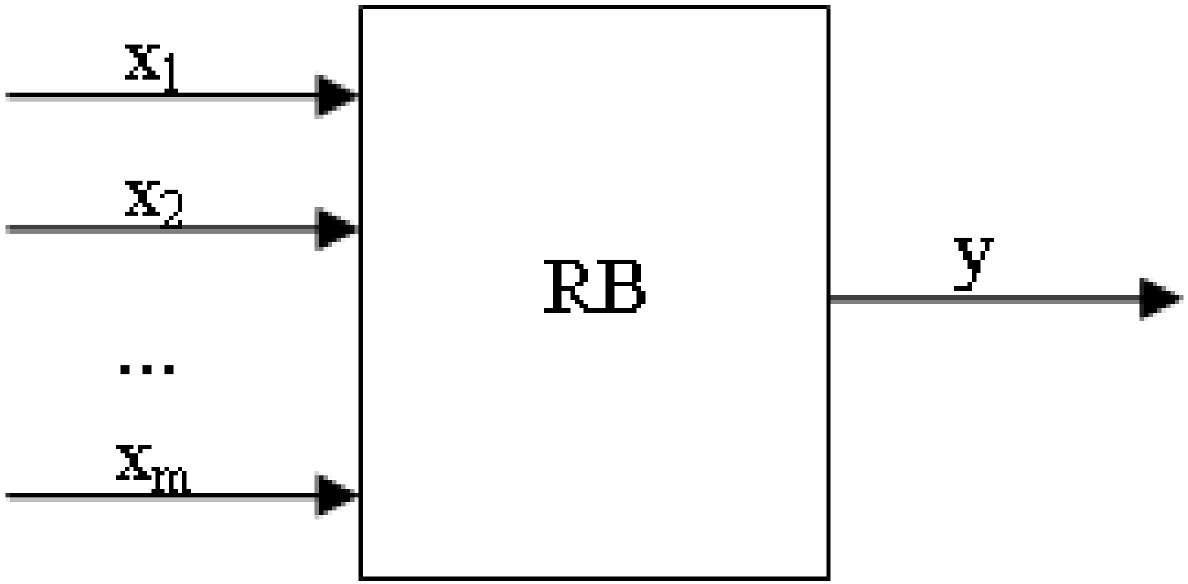


Figure 1: Standard rule based system (single rule base)

Types of Rule Based Systems

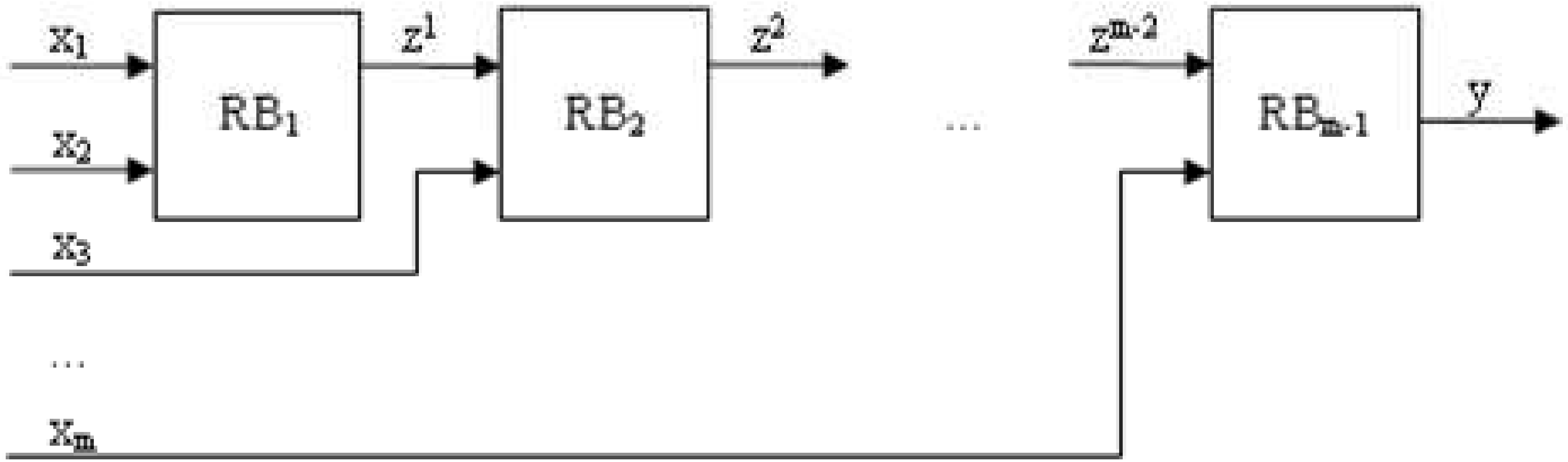


Figure 2: Hierarchical rule based system (chained rule bases)

Types of Rule Based Systems

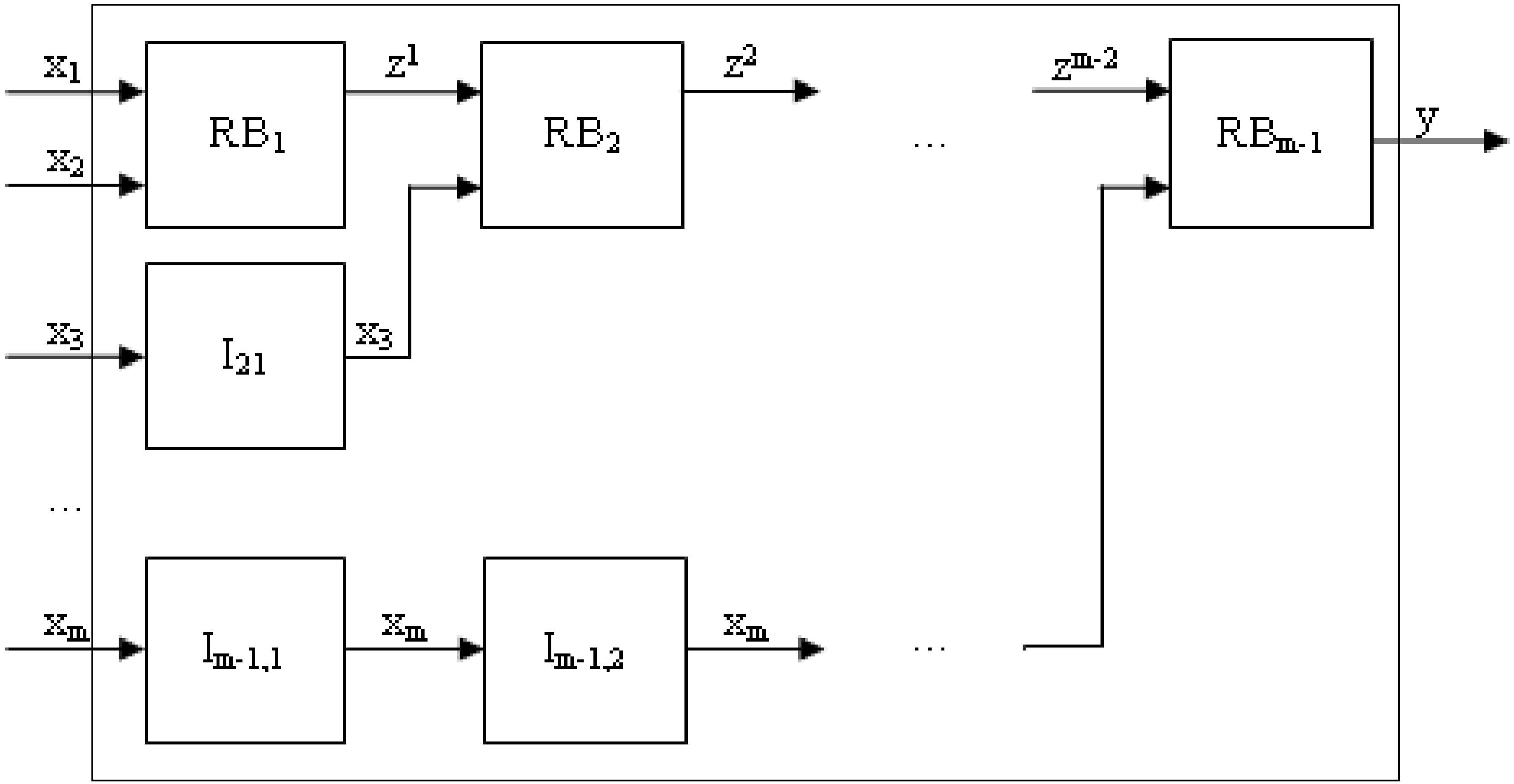


Figure 3: Rule based network (modular rule bases)

Formal Models for Rule Based Networks

Node Modelling by If-then Rules

Rule 1: If x is low, then y is small
Rule 2: If x is average, then y is medium
Rule 3: If x is high, then y is big

Node Modelling by Integer Tables

Linguistic terms for x	Linguistic terms for y
1 (low)	1 (small)
2 (average)	2 (medium)
3 (high)	3 (big)

Formal Models for Rule Based Networks

Node Modelling by Boolean Matrices

x / y	1 (small)	2 (medium)	3 (big)
1 (low)	1	0	0
2 (average)	0	1	0
3 (high)	0	0	1

Node Modelling by Binary Relations

{(1, 1), (2, 2), (3, 3)}

Formal Models for Rule Based Networks

Network Modelling by Grid Structures

Layer 1

Level 1 $N_{11}(x, y)$

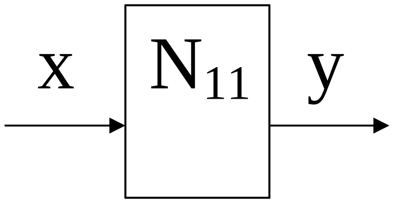
Network Modelling by Interconnection Structures

Layer 1

Level 1 y

Formal Models for Rule Based Networks

Network Modelling by Block Schemes



Network Modelling by Topological Expressions

$[N_{11}]\ (x \mid y)$

Basic Operations in Rule Based Networks

Horizontal Merging of Nodes

$[N_{11}]\ (x_{11}\,|\,z_{11,12}) * [N_{12}]\ (z_{11,12}\,|\,y_{12}) = [N_{11*12}]\ (x_{11}\,|\,y_{12})$

* symbol for horizontal merging

N_{11} :

x_{11} \ $z_{11,12}$	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

Basic Operations in Rule Based Networks

N_{12} :

$z_{11,12}$ \ y_{12}	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

N_{11*12} :

x_{11} \ y_{12}	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Basic Operations in Rule Based Networks

Horizontal Splitting of Nodes

$[N_{11/12}]\ (x_{11} \mid y_{12}) = [N_{11}]\ (x_{11} \mid z_{11,12})\ /\ [N_{12}]\ (z_{11,12} \mid y_{12})$

/ symbol for horizontal splitting

$N_{11/12}$:

x_{11} \ y_{12}	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

Basic Operations in Rule Based Networks

N_{11} :

x_{11} \ $z_{11,12}$	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

N_{12} :

$z_{11,12}$ \ y_{12}	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

Basic Operations in Rule Based Networks

<u>Vertical Merging of Nodes</u>

$$[N_{11}]\ (x_{11} \mid y_{11}) + [N_{21}]\ (x_{21} \mid y_{21}) = [N_{11+21}]\ (x_{11}, x_{21} \mid y_{11}, y_{21})$$

\+ symbol for vertical merging

Basic Operations in Rule Based Networks

N_{11} :

x_{11} \ y_{11}	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

N_{21} :

x_{21} \ y_{21}	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Basic Operations in Rule Based Networks

N_{11+21} :

x_{11}, x_{21} \ y_{11}, y_{21}	11	12	13	21	22	23	31	32	33
11	0	1	0	0	0	0	0	0	0
12	1	0	0	0	0	0	0	0	0
13	0	0	1	0	0	0	0	0	0
21	0	0	0	0	0	0	0	1	0
22	0	0	0	0	0	0	1	0	0
23	0	0	0	0	0	0	0	0	1
31	0	0	0	0	1	0	0	0	0
32	0	0	0	1	0	0	0	0	0
33	0	0	0	0	0	1	0	0	0

Basic Operations in Rule Based Networks

Vertical Splitting of Nodes

$$[N_{11-21}]\ (x_{11}, x_{21} \,|\, y_{11}, y_{21}) = [N_{11}]\ (x_{11} \,|\, y_{11}) - [N_{21}]\ (x_{21} \,|\, y_{21})$$

– symbol for vertical splitting

Basic Operations in Rule Based Networks

N_{11+21} :

x_{11}, x_{21} \ y_{11}, y_{21}	11	12	13	21	22	23	31	32	33
11	0	0	0	0	0	1	0	0	0
12	0	0	0	1	0	0	0	0	0
13	0	0	0	0	1	0	0	0	0
21	0	0	0	0	0	0	0	0	1
22	0	0	0	0	0	0	1	0	0
23	0	0	0	0	0	0	0	1	0
31	0	0	1	0	0	0	0	0	0
32	1	0	0	0	0	0	0	0	0
33	0	1	0	0	0	0	0	0	0

Basic Operations in Rule Based Networks

N_{11} :

x_{11} \ y_{11}	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

N_{21} :

x_{21} \ y_{21}	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

Basic Operations in Rule Based Networks

Output Merging of Nodes

$[N_{11}]\ (x_{11,21} \,|\, y_{11})\ ;\ [N_{21}]\ (x_{11,21} \,|\, y_{21}) = [N_{11;21}]\ (x_{11,21} \,|\, y_{11}, y_{21})$

; symbol for output merging

N_{11} :

$x_{11,21}$ \ y_{11}	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

Basic Operations in Rule Based Networks

N_{21} :

$x_{11,21}$ \ y_{21}	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

$N_{11;21}$:

$x_{11,21}$ \ y_{11}, y_{21}	11	12	13	21	22	23	31	32	33
1	0	1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	1	0	0
3	0	0	0	0	0	1	0	0	0

Basic Operations in Rule Based Networks

Output Splitting of Nodes

$[N_{11:21}]\ (x_{11,\ 21} \,|\, y_{11}, y_{21}) = [N_{11}]\ (x_{11,21} \,|\, y_{11}) : [N_{21}]\ (x_{11,21} \,|\, y_{21})$

: symbol for output splitting

$N_{11:21}$:

$x_{11,21}$ \ y_{11}, y_{21}	11	12	13	21	22	23	31	32	33
1	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	1	0	0
3	0	1	0	0	0	0	0	0	0

Basic Operations in Rule Based Networks

N_{11} :

$x_{11,21}$ \ y_{11}	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

N_{21} :

$x_{11,21}$ \ y_{21}	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

Structural Properties of Basic Operations

<u>Associativity of Horizontal Merging</u>

$[N_{11}]\ (x_{11} \mid z_{11,12}) * [N_{12}]\ (z_{11,12} \mid z_{12,13}) * [N_{13}]\ (z_{12,13} \mid y_{13}) =$

$[N_{11*12}]\ (x_{11} \mid z_{12,13}) * [N_{13}]\ (z_{12,13} \mid y_{13}) =$

$[N_{11}]\ (x_{11} \mid z_{11,12}) * [N_{12*13}]\ (z_{11,12} \mid y_{13}) =$

$[N_{11*12*13}]\ (x_{11} \mid y_{13})$

Structural Properties of Basic Operations

N_{11} :

$z_{11,12}$ / x_{11}	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

N_{12} :

$z_{12,13}$ / $z_{11,12}$	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

Structural Properties of Basic Operations

N_{13} :

$z_{12,13}$ \ y_{13}	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

$N_{(11*12)*13} = N_{11*(12*13)}$:

x_{11} \ y_{13}	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

Structural Properties of Basic Operations

Variability of Horizontal Splitting

$[N_{11/12/13}]\ (x_{11} \mid y_{13}) =$

$[N_{11}]\ (x_{11} \mid z_{11,12})\ /\ [N_{12/13}]\ (z_{11,12} \mid y_{13}) =$

$[N_{11/12}]\ (x_{11} \mid z_{12,13})\ /\ [N_{13}]\ (z_{12,13} \mid y_{13}) =$

$[N_{11}]\ (x_{11} \mid z_{11,12})\ /\ [N_{12}]\ (z_{11,12} \mid z_{12,13})\ /\ [N_{13}]\ (z_{12,13} \mid y_{13})$

Structural Properties of Basic Operations

$N_{11/(12/13)} = N_{(11/21)/31}$:

x_{11} \ y_{13}	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

N_{11} :

x_{11} \ $z_{11,12}$	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Structural Properties of Basic Operations

N_{12} :

$z_{11,12}$ \ $z_{12,13}$	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

N_{13} :

$z_{12,13}$ \ y_{13}	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

Structural Properties of Basic Operations

Associativity of Vertical Merging

$[N_{11}]\ (x_{11} \mid y_{11}) + [N_{21}]\ (x_{21} \mid y_{21}) + [N_{31}]\ (x_{31} \mid y_{31}) =$

$[N_{11+21}]\ (x_{11}, x_{21} \mid y_{11}, y_{21}) + [N_{31}]\ (x_{31} \mid y_{31}) =$

$[N_{11}]\ (x_{11} \mid y_{11}) + [N_{21+31}]\ (x_{21}, x_{31} \mid y_{21}, y_{31}) =$

$[N_{11+21+31}]\ (x_{11}, x_{21}, x_{31} \mid y_{11}, y_{21}, y_{31})$

Structural Properties of Basic Operations

N_{11} :

x_{11} \ y_{11}	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

N_{21} :

x_{21} \ y_{21}	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Structural Properties of Basic Operations

N_{31} :

x_{31} \ y_{13}	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

Structural Properties of Basic Operations

$N_{(11+21)+31} = N_{11+(21+31)}$: y_{11}, y_{21}, y_{31} / x_{11}, x_{21}, x_{31}

x_{11}, x_{21}, x_{31}	A	B	C	D	E	F	G	H	I
A (111-113)	0_3	1_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3
B (121-123)	1_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3
C (131-133)	0_3	0_3	1_3	0_3	0_3	0_3	0_3	0_3	0_3
D (211-213)	0_3	0_3	0_3	0_3	0_3	0_3	0_3	1_3	0_3
E (221-223)	0_3	0_3	0_3	0_3	0_3	0_3	1_3	0_3	0_3
F (231-133)	0_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3	1_3
G (311-313)	0_3	0_3	0_3	0_3	1_3	0_3	0_3	0_3	0_3
H (321-323)	0_3	0_3	0_3	1_3	0_3	0_3	0_3	0_3	0_3
I (331-333)	0_3	0_3	0_3	0_3	0_3	1_3	0_3	0_3	0_3

0_3 = ‘3 by 3’ null matrix, 1_3 = ‘3 by 3’ anti-diagonal matrix

Structural Properties of Basic Operations

Variability of Vertical Splitting

$[N_{11-21-31}]\ (x_{11}, x_{21}, x_{31} \mid y_{11}, y_{21}, y_{31}) =$

$[N_{11}]\ (x_{11} \mid y_{11}) - [N_{21-31}]\ (x_{21}, x_{31} \mid y_{21}, y_{31}) =$

$[N_{11-21}]\ (x_{11}, x_{21} \mid y_{11}, y_{21}) - [N_{31}]\ (x_{31} \mid y_{31}) =$

$[N_{11}]\ (x_{11} \mid y_{11}) - [N_{21}]\ (x_{21} \mid y_{21}) - [N_{31}]\ (x_{31} \mid y_{31})$

Structural Properties of Basic Operations

$N_{11-(21-31)} = N_{(11-21)-31}$: y_{11}, y_{21}, y_{31} / x_{11}, x_{21}, x_{31}

x_{11}, x_{21}, x_{31}	A	B	C	D	E	F	G	H	I
A (111-113)	0_3	0_3	0_3	0_3	0_3	1_3	0_3	0_3	0_3
B (121-123)	0_3	0_3	0_3	1_3	0_3	0_3	0_3	0_3	0_3
C (131-133)	0_3	0_3	0_3	0_3	1_3	0_3	0_3	0_3	0_3
D (211-213)	0_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3	1_3
E (221-223)	0_3	0_3	0_3	0_3	0_3	0_3	1_3	0_3	0_3
F (231-133)	0_3	0_3	0_3	0_3	0_3	0_3	0_3	1_3	0_3
G (311-313)	0_3	0_3	1_3	0_3	0_3	0_3	0_3	0_3	0_3
H (321-323)	1_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3
I (331-333)	0_3	1_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3

0_3 = '3 by 3' null matrix, 1_3 = '3 by 3' anti-diagonal matrix

Structural Properties of Basic Operations

N_{11} :

x_{11} \ y_{11}	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

N_{21} :

x_{21} \ y_{21}	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

Structural Properties of Basic Operations

N_{31} :

x_{31} \ y_{13}	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

Structural Properties of Basic Operations

Associativity of Output Merging

$[N_{11}]\ (x_{11,21,31} \mid y_{11})\ ;\ [N_{21}]\ (x_{11,21,31} \mid y_{21})\ ;\ [N_{31}]\ (x_{11,21,31} \mid y_{31}) =$

$[N_{11;21}]\ (x_{11,21,31} \mid y_{11}, y_{21})\ ;\ [N_{31}]\ (x_{11,21,31} \mid y_{31}) =$

$[N_{11}]\ (x_{11,21,31} \mid y_{11})\ ;\ [N_{21;31}]\ (x_{11,21,31} \mid y_{21}, y_{31}) =$

$[N_{11;21;31}]\ (x_{11,21,31} \mid y_{11}, y_{21}, y_{31})$

Structural Properties of Basic Operations

N_{11} :

y_{11} / $x_{11,21,31}$	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

N_{21} :

y_{21} / $x_{11,21,31}$	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Structural Properties of Basic Operations

N_{31} :

$x_{11,21,31}$ \ y_{13}	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

$N_{(11;21);31} = N_{11;(21;31)}$:

$x_{11,21,31}$ \ y_{11}, y_{21}, y_{31}	A	B	C	D	E	F	G	H	I
1	0_3	1_3	0_3	0_3	0_3	0_3	0_3	0_3	0_3
2	0_3	0_3	0_3	0_3	0_3	0_3	1_2	0_3	0_3
3	0_3	0_3	0_3	0_3	0_3	1_1	0_3	0_3	0_3

0_3 = 0 0 0, 1_3 = 0 0 1, 1_2 = 0 1 0, 1_1 = 1 0 0

Structural Properties of Basic Operations

Variability of Output Splitting

$[N_{11:21:31}]\ (x_{11,21,31} \mid y_{11}, y_{21}, y_{31}) =$

$[N_{11}]\ (x_{11,21,31} \mid y_{11}) : [N_{21:31}]\ (x_{11,21,31} \mid y_{21}, y_{31}) =$

$[N_{11:21}]\ (x_{11,21,31} \mid y_{11}, y_{21}) : [N_{31}]\ (x_{11,21,31} \mid y_{31}) =$

$[N_{11}]\ (x_{11,21,31} \mid y_{11}) : [N_{21}]\ (x_{11,21,31} \mid y_{21}) : [N_{31}]\ (x_{11,21,31} \mid y_{31})$

Structural Properties of Basic Operations

$N_{11:(21:31)} = N_{(11:21):31} : y_{11}, y_{21}, y_{31}$

$x_{11,21,31}$	A	B	C	D	E	F	G	H	I
1	0_3	0_3	0_3	0_3	1_3	0_3	0_3	0_3	0_3
2	0_3	0_3	0_3	0_3	0_3	0_3	1_2	0_3	0_3
3	0_3	1_1	0_3	0_3	0_3	0_3	0_3	0_3	0_3

$0_3 = 0\ 0\ 0,\ 1_3 = 0\ 0\ 1,\ 1_2 = 0\ 1\ 0,\ 1_1 = 1\ 0\ 0$

N_{11} :

$x_{11,12,13}$ \ y_{11}	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

Structural Properties of Basic Operations

N_{21} :

$x_{11,21,31}$ \ y_{21}	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

N_{31} :

$x_{11,21,31}$ \ y_{31}	1	2	3
1	0	0	1
2	0	1	0
3	1	0	0

Advanced Operations in Rule Based Networks

Node Transformation in Input Augmentation

$$[N]\,(x \mid y) \Rightarrow [N^{AI}]\,(x\,,\,x^{AI} \mid y)$$

N :

x \ y	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Advanced Operations in Rule Based Networks

N^{AI} :

x, x^{AI} \ y	1	2	3
11	0	1	0
12	0	1	0
13	0	1	0
21	1	0	0
22	1	0	0
23	1	0	0
31	0	0	1
32	0	0	1
33	0	0	1

Advanced Operations in Rule Based Networks

Node Transformation in Output Permutation

$[N]\ (x \mid y_1, y_2) \Rightarrow [N^{PO}]\ (x \mid y_2, y_1)$

N :

x \ y_1, y_2	11	12	13	21	22	23	31	32	33
1	0	0	0	0	0	1	0	0	0
2	0	0	0	0	0	0	1	0	0
3	0	1	0	0	0	0	0	0	0

Advanced Operations in Rule Based Networks

N^{PO} :

x \ y_2, y_1	11	12	13	21	22	23	31	32	33
1	0	0	0	0	0	0	0	1	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0

Advanced Operations in Rule Based Networks

<u>Node Transformation in Feedback Equivalence</u>

$$[N]\,(x, z \,|\, y, z) \Rightarrow [N^{EF}]\,(x, x^{EF} \,|\, y, y^{EF})$$

N :

x, z \ y, z	11	12	21	22
11	?	?	?	?
12	?	?	?	?
21	?	?	?	?
22	?	?	?	?

Advanced Operations in Rule Based Networks

N^{EF}:

x, x^{EF} \ y, y^{EF}	11	12	21	22
11	1	0	1	0
12	0	1	0	1
21	1	0	1	0
22	0	1	0	1

Advanced Operations in Rule Based Networks

Node Identification in Horizontal Merging

A * U = C

A, C – known nodes, U – non-unique unknown node

A :

x_{1A}, x_{2A} \ $z_{A,U}$	1	2	3
11	1	0	0
12	1	0	0
21	0	0	1
22	0	0	1

Advanced Operations in Rule Based Networks

C :

x_{1A}, x_{2A} \ y_{1U}, y_{2U}	11	12	21	22
11	0	0	0	1
12	0	0	0	1
21	1	0	0	0
22	1	0	0	0

U:

$z_{A,U}$ \ y_{1U}, y_{2U}	11	12	21	22
1	0	0	0	1
2	0	0	0	0
3	1	0	0	0

Advanced Operations in Rule Based Networks

U * B = C

B, C – known nodes, U – non-unique unknown node

B :

$z_{U,B}$ \ y_{1B}, y_{2B}	11	12	21	22
1	1	0	0	0
2	0	0	0	0
3	0	0	0	1

Advanced Operations in Rule Based Networks

C :

x_{1U}, x_{2U} \ y_{1B}, y_{2B}	11	12	21	22
11	0	0	0	1
12	0	0	0	1
21	1	0	0	0
22	1	0	0	0

U :

x_{1U}, x_{2U} \ $z_{U,B}$	1	2	3
11	0	0	1
12	0	0	1
21	1	0	0
22	1	0	0

Advanced Operations in Rule Based Networks

A * U * B = C

A, B, C – known nodes, U – non-unique unknown node

A * U = D
D * B = C

D – non-unique unknown node

U * B = E
A * E = C

E – non-unique unknown node

Advanced Operations in Rule Based Networks

Node Identification in Vertical Merging

A + U = C

A, C – known nodes, U – unique known node

A :

x_A \ y_A	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Advanced Operations in Rule Based Networks

C :

x_A, x_U \ y_A, y_U	11	12	13	21	22	23	31	32	33
11	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	1	0	0	0
13	0	0	0	0	1	0	0	0	0
21	1	0	0	0	0	0	0	0	0
22	0	0	1	0	0	0	0	0	0
23	0	1	0	0	0	0	0	0	0
31	0	0	0	0	0	0	1	0	0
32	0	0	0	0	0	0	0	0	1
33	0	0	0	0	0	0	0	1	0

Advanced Operations in Rule Based Networks

U :

x_U \ y_U	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

Advanced Operations in Rule Based Networks

U + B = C

B, C – known nodes, U – unique unknown node

B :

x_B \ y_B	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

Advanced Operations in Rule Based Networks

C :

x_U, x_B \ y_U, y_B	11	12	13	21	22	23	31	32	33
11	0	0	0	0	0	0	0	1	0
12	0	0	0	0	0	0	0	0	1
13	0	0	0	0	0	0	1	0	0
21	0	1	0	0	0	0	0	0	0
22	0	0	1	0	0	0	0	0	0
23	1	0	0	0	0	0	0	0	0
31	0	0	0	0	1	0	0	0	0
32	0	0	0	0	0	1	0	0	0
33	0	0	0	1	0	0	0	0	0

Advanced Operations in Rule Based Networks

U :

x_U \ y_U	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

Advanced Operations in Rule Based Networks

A + U + B = C

A, B, C – known nodes, U – unique unknown node

A + U = D
D + B = C

D – unique unknown node

U + B = E
A + E = C

E – unique unknown node

Advanced Operations in Rule Based Networks

Node Identification in Output Merging

A ; U = C

A, C – known nodes, U – unique unknown node

A :

$x_{A,U}$ \ y_A	1	2	3
1	0	1	0
2	1	0	0
3	0	0	1

Advanced Operations in Rule Based Networks

C :

$x_{A,U}$ \ y_A, y_U	11	12	13	21	22	23	31	32	33
1	0	0	0	1	0	0	0	0	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	0	0	0	0	1	0

U :

$x_{A,U}$ \ y_U	1	2	3
1	1	0	0
2	0	0	1
3	0	1	0

Advanced Operations in Rule Based Networks

U ; B = C

B, C – known nodes, U – unique unknown node

B :

$x_{U,B}$ \ y_B	1	2	3
1	0	1	0
2	0	0	1
3	1	0	0

Advanced Operations in Rule Based Networks

C :

$x_{U,B}$ \ y_U, y_B	11	12	13	21	22	23	31	32	33
1	0	0	0	0	0	0	0	1	0
2	0	0	1	0	0	0	0	0	0
3	0	0	0	1	0	0	0	0	0

U :

$x_{U,B}$ \ y_U	1	2	3
1	0	0	1
2	1	0	0
3	0	1	0

Advanced Operations in Rule Based Networks

A ; U ; B = C

A, B, C – known nodes, U – unique unknown node

A ; U = D
D ; B = C

D – unique unknown node

U ; B = E
A ; E = C

E – unique unknown node

Feedforward Rule Based Networks

Network with Single Level and Single Layer

- rule based system

1,1

$[N_{11}]$ $(x_{11} \mid y_{11})$

Feedforward Rule Based Networks

<u>Network with Single Level and Multiple Layers</u>

- queue of 'n' rule based systems

1,1	…	1,n

Example 1 (Network Analysis)

$$[N_{11}]\ (x_{11}\ |\ z_{11,12}^{\ 1,2},\ z_{11,12}^{\ 2,1})\ *\ [N_{12}]\ (z_{11,12}^{\ 2,1},\ z_{11,12}^{\ 1,2}\ |\ y_{12}) \Rightarrow$$

$$[N_{11}^{\ PO}]\ (x_{11}\ |\ z_{11,12}^{\ 2,1},\ z_{11,12}^{\ 1,2})\ *\ [N_{12}]\ (z_{11,12}^{\ 2,1},\ z_{11,12}^{\ 1,2}\ |\ y_{12}) \Rightarrow$$

$$[N_{11}^{\ PO} * N_{12}]\ (x_{11}\ |\ y_{12}) \Rightarrow$$

Feedforward Rule Based Networks

$N_E = N_{11}^{PO} * N_{12}$

Algorithm 1.1 (Network Design)

1. Define N_E and N_{12}.
2. Derive N_{11}^{PO}, if possible.
3. Find N_{11} by inverse output permutation of N_{11}^{PO}.

Algorithm 1.2 (Network Design)

1. Define N_E and N_{11}.
2. Find N_{11}^{PO} by output permutation of N_{11}.
3. Derive N_{12}, if possible.

Feedforward Rule Based Networks

<u>Network with Multiple Levels and Single Layer</u>

- stack of 'm' rule based systems

1,1
…
m,1

Example 2 (Network Analysis)

$[N_{11}]\ (x_{11,21}^{1,1}, x_{11}^{2} \mid y_{11})\ ;\ [N_{21}]\ (x_{11,21}^{1,1} \mid y_{21}) \Rightarrow$

$[N_{11}]\ (x_{11,21}^{1,1}, x_{11}^{2} \mid y_{11})\ ;\ [N_{21}^{AI}]\ (x_{11,21}^{1,1}, x_{11}^{2} \mid y_{21}) \Rightarrow$

Feedforward Rule Based Networks

$[N_{11} ; N_{21}^{AI}] (x_{11,21}^{1,1}, x_{11}^{2} \mid y_{11}, y_{21}) \Rightarrow$

$N_E = N_{11} ; N_{21}^{AI}$

Algorithm 2.1 (Network Design)

1. Define N_E and N_{21}.
2. Find N_{21}^{AI} by input augmentation of N_{21}.
3. Derive N_{11}, if possible.

Algorithm 2.2 (Network Design)

1. Define N_E and N_{11}.
2. Derive N_{21}^{AI}, if possible.
3. Find N_{21} by inverse input augmentation of N_{21}^{AI}.

Feedforward Rule Based Networks

<u>Network with Multiple Levels and Multiple Layers</u>

- grid of 'm by n' rule based systems

1,1	…	1,n
…	…	…
m,1	…	m,n

Example 3 (Network Analysis)

$$\{[N_{11}]\ (x_{11} \mid z_{11,12}{}^{1,1},\ z_{11,22}{}^{2,1}) + [N_{21}]\ (x_{21} \mid z_{21,12}{}^{1,2})\} \ *$$

$$\{[N_{12}]\ (z_{11,12}{}^{1,1},\ z_{21,12}{}^{1,2} \mid y_{12}) + [N_{22}]\ (z_{11,22}{}^{2,1} \mid y_{22})\} \Rightarrow$$

Feedforward Rule Based Networks

$$[N_{11} + N_{21}]\ (x_{11}, x_{21} \mid z_{11,12}^{1,1}, z_{11,22}^{2,1}, z_{21,12}^{1,2})\ *$$

$$[N_{12} + N_{22}]\ (z_{11,12}^{1,1}, z_{21,12}^{1,2}, z_{11,22}^{2,1} \mid y_{12}, y_{22}) \Rightarrow$$

$$[(N_{11} + N_{21})^{PO}]\ (x_{11}, x_{21} \mid z_{11,12}^{1,1}, z_{21,12}^{1,2}, z_{11,22}^{2,1})\ *$$

$$[N_{12} + N_{22}]\ (z_{11,12}^{1,1}, z_{21,12}^{1,2}, z_{11,22}^{2,1} \mid y_{12}, y_{22}) \Rightarrow$$

$$[(N_{11} + N_{21})^{PO} * (N_{12} + N_{22})]\ (x_{11}, x_{21} \mid y_{12}, y_{22}) \Rightarrow$$

$$N_E = (N_{11} + N_{21})^{PO} * (N_{12} + N_{22})$$

Feedforward Rule Based Networks

Algorithm 3.1 (Network Design)

1. Define N_E, N_{21}, N_{12} and N_{22}.
2. Find $N_{12} + N_{22}$ by vertical merging of $N_{12} + N_{22}$.
3. Derive $(N_{11} + N_{21})^{PO}$, if possible.
4. Find $N_{11} + N_{21}$ by inverse output permutation of $(N_{11} + N_{21})^{PO}$.
5. Derive N_{11} from $N_{11} + N_{21}$, if possible.

Algorithm 3.2 (Network Design)

1. Define N_E, N_{11}, N_{12} and N_{22}.
2. Find $N_{12} + N_{22}$ by vertical merging of $N_{12} + N_{22}$.
3. Derive $(N_{11} + N_{21})^{PO}$, if possible.
4. Find $N_{11} + N_{21}$ by inverse output permutation of $(N_{11} + N_{21})^{PO}$.
5. Derive N_{21} from $N_{11} + N_{21}$, if possible.

Feedforward Rule Based Networks

Algorithm 3.3 (Network Design)

1. Define N_E, N_{11}, N_{21} and N_{22}.
2. Find $N_{11} + N_{21}$ by vertical merging of $N_{11} + N_{21}$.
3. Find $(N_{11} + N_{21})^{PO}$ by output permutation of $N_{11} + N_{21}$.
4. Derive $N_{12} + N_{22}$, if possible.
5. Derive N_{12} from $N_{12} + N_{22}$, if possible.

Algorithm 3.4 (Network Design)

1. Define N_E, N_{11}, N_{21} and N_{12}.
2. Find $N_{11} + N_{21}$ by vertical merging of $N_{11} + N_{21}$.
3. Find $(N_{11} + N_{21})^{PO}$ by output permutation of $N_{11} + N_{21}$.
4. Derive $N_{12} + N_{22}$, if possible.
5. Derive N_{22} from $N_{12} + N_{22}$, if possible.

Feedback Rule Based Networks

<u>Network with Single Local Feedback</u>

- one node embraced by feedback

N(F)	N
N	N

N	N(F)
N	N

N	N
N(F)	N

N	N
N	N(F)

N(F) – node embraced by feedback
N – nodes with no feedback

Feedback Rule Based Networks

Example 4 (Network Analysis)

$[N_{11}]\ (x_{11}, w_{11}\ |\ z_{11,12}{}^{2,1}, v_{11}) * [N_{12}]\ (z_{11,12}{}^{1,1}\ |\ y_{12}),$

$[F_{11}]\ (v_{11}\ |\ w_{11}) \Rightarrow$

$[N_{11}]\ (x_{11}, w_{11}\ |\ z_{11,12}{}^{2,1}, v_{11}) *$

$\{[N_{12}]\ (z_{11,12}{}^{1,1}\ |\ y_{12}) + [F_{11}]\ (v_{11}\ |\ w_{11})\} \Rightarrow$

$[N_{11} * (N_{12} + F_{11})]\ (x_{11}, w_{11}\ |\ y_{12}, w_{11}) \Rightarrow$

$[(N_{11} * (N_{12} + F_{11}))^{EF}]\ (x_{11}, x^{EF}\ |\ y_{12}, y^{EF}) \Rightarrow$

Feedback Rule Based Networks

$N_E = (N_{11} * (N_{12} + F_{11}))^{EF}$

F_{11} – feedback node

Algorithm 4.1 (Network Design)

1. Define N_E, N_{11} and N_{12}.
2. Confirm that N_E satisfies the feedback constraints, if possible.
3. Make N_E equal to $N_{11} * (N_{12} + F_{11})$.
4. Derive $N_{12} + F_{11}$ from N_E, if possible.
5. Derive F_{11} from $N_{12} + F_{11}$, if possible.

Feedback Rule Based Networks

<u>Network with Multiple Local Feedback</u>

- at least two nodes embraced by separate feedback

N(F1)	N(F2)
N	N

N	N
N(F1)	N(F2)

N(F1)	N
N	N(F2)

N(F1)	N
N(F2)	N

N	N(F1)
N	N(F2)

N	N(F1)
N(F2)	N

N(F1), N(F2) – nodes embraced by separate feedback
N – nodes with no feedback

Feedback Rule Based Networks

Example 5 (Network Analysis)

$[N_{11}]\ (x_{11}, w_{11} \,|\, z_{11,12}{}^{1,1}, v_{11}) * [N_{12}]\ (z_{11,12}{}^{1,1}, w_{12} \,|\, y_{12}, v_{12}),$

$[F_{11}]\ (v_{11} \,|\, w_{11}),\ [F_{12}]\ (v_{12} \,|\, w_{12}) \Rightarrow$

$\{[N_{11}]\ (x_{11}, w_{11} \,|\, z_{11,12}{}^{1,1}, v_{11}) + [I_{31}]\ (w_{12} \,|\, w_{12})\} *$

$\{[N_{12}]\ (z_{11,12}{}^{1,1}, w_{12} \,|\, y_{12}, v_{12}) + [F_{11}]\ (v_{11} \,|\, w_{11})\} *$

$\{[I_{13}]\ (y_{12} \,|\, y_{12}) + [F_{12}]\ (v_{12} \,|\, w_{12}) + [I_{33}]\ (w_{11} \,|\, w_{11})\} \Rightarrow$

Feedback Rule Based Networks

$[N_{11} + I_{31}]\ (x_{11}, w_{11}, w_{12} \mid z_{11,12}{}^{1,1}, v_{11}, w_{12})$ *

$[N_{12} + F_{11}]\ (z_{11,12}{}^{1,1}, w_{12}, v_{11} \mid y_{12}, v_{12}, w_{11})$ *

$[I_{13} + F_{12} + I_{33}]\ (y_{12}, v_{12}, w_{11} \mid y_{12}, w_{12}, w_{11}) \Rightarrow$

$[(N_{11} + I_{31})^{PO}]\ (x_{11}, w_{11}, w_{12} \mid z_{11,12}{}^{1,1}, w_{12}, v_{11})$ *

$[N_{12} + F_{11}]\ (z_{11,12}{}^{1,1}, w_{12}, v_{11} \mid y_{12}, v_{12}, w_{11})$ *

$[I_{13} + F_{12} + I_{33}]\ (y_{12}, v_{12}, w_{11} \mid y_{12}, w_{12}, w_{11}) \Rightarrow$

Feedback Rule Based Networks

$$[(N_{11} + I_{31})^{PO} * (N_{12} + F_{11}) * (I_{13} + F_{12} + I_{33})]$$

$$(x_{11}, w_{11}, w_{12} \mid y_{12}, w_{12}, w_{11}) \Rightarrow$$

$$[((N_{11} + I_{31})^{PO} * (N_{12} + F_{11}) * (I_{13} + F_{12} + I_{33}))^{FE}]$$

$$(x_{11}, x_1^{FE}, x_2^{FE} \mid y_{12}, y_2^{FE}, y_1^{FE}) \Rightarrow$$

$$N_E = ((N_{11} + I_{31})^{PO} * (N_{12} + F_{11}) * (I_{13} + F_{12} + I_{33}))^{FE}$$

F_{11}, F_{12} – feedback nodes
I_{31}, I_{13}, I_{33} – identity nodes

Feedback Rule Based Networks

Algorithm 5.1 (Network Design)

1. Define N_E, N_{11}, N_{12}, I_{31}, I_{13}, I_{33} and F_{12}.
2. Confirm that N_E satisfies the feedback constraints, if possible.
3. Make N_E equal to $(N_{11} + I_{31})^{PO} * (N_{12} + F_{11}) * (I_{13} + F_{12} + I_{33})$.
4. Find $N_{11} + I_{31}$ by vertical merging of N_{11} and I_{31}.
5. Find $(N_{11} + I_{31})^{PO}$ by output permutation of $N_{11} + I_{31}$.
6. Find $I_{13} + F_{12} + I_{33}$ by vertical merging of I_{13}, F_{12} and I_{33}.
7. Derive $N_{12} + F_{11}$ from N_E, if possible.
8. Derive F_{11} from $N_{12} + F_{11}$, if possible.

Feedback Rule Based Networks

Algorithm 5.2 (Network Design)

1. Define N_E, N_{11}, N_{12}, I_{31}, I_{13}, I_{33} and F_{11}.
2. Confirm that N_E satisfies the feedback constraints, if possible.
3. Make N_E equal to $(N_{11} + I_{31})^{PO} * (N_{12} + F_{11}) * (I_{13} + F_{12} + I_{33})$.
4. Find $N_{11} + I_{31}$ by vertical merging of N_{11} and I_{31}.
5. Find $(N_{11} + I_{31})^{PO}$ by output permutation of $N_{11} + I_{31}$.
6. Find $N_{12} + F_{11}$ by vertical merging of N_{12} and F_{11}.
7. Find $(N_{11} + I_{31})^{PO} * (N_{12} + F_{11})$ by horizontal merging of $(N_{11} + I_{31})^{PO}$ and $(N_{12} + F_{11})$.
8. Derive $I_{13} + F_{12} + I_{33}$ from N_E, if possible.
9. Derive F_{12} from $I_{13} + F_{12} + I_{33}$, if possible.

Feedback Rule Based Networks

<u>Network with Single Global Feedback</u>

- one set of at least two adjacent nodes embraced by feedback

N1(F)	N2(F)
N	N

N	N
N1(F)	N2(F)

N1(F)	N
N2(F)	N

N	N1(F)
N	N2(F)

{N1(F), N2(F)} – set of nodes embraced by feedback
N – nodes with no feedback

Feedback Rule Based Networks

Example 6 (Network Analysis)

$[N_{11}]\ (x_{11}, w_{12,11} \mid z_{11,12}^{1,1}) * [N_{12}]\ (z_{11,12}^{1,1} \mid y_{12}, v_{12,11}),$

$[F_{12,11}]\ (v_{12,11} \mid w_{12,11}) \Rightarrow$

$[N_{11}]\ (x_{11}, w_{12,11} \mid z_{11,12}^{1,1}) * [N_{12}]\ (z_{11,12}^{1,1} \mid y_{12}, v_{12,11}) *$

$\{[I_{13}]\ (y_{12} \mid y_{12}) + [F_{12,11}]\ (y_{12}, v_{12,11} \mid y_{12}, w_{12,11})\} \Rightarrow$

$[N_{11} * N_{12} * (I_{13} + F_{12,11})]\ (x_{11}, w_{12,11} \mid y_{12}, w_{12,11})$

$[(N_{11} * N_{12} * (I_{13} + F_{12,11}))^{EF}]\ (x_{11}, x^{EF} \mid y_{12}, y^{EF}) \Rightarrow$

Feedback Rule Based Networks

$N_E = (N_{11} * N_{12} * (I_{13} + F_{12,11}))^{EF}$

$F_{12,11}$ – feedback node
I_{13} – identity node

Algorithm 6.1 (Network Design)

1. Define N_E, N_{11}, N_{12}, I_{13} and $F_{12,11}$.
2. Confirm that N_E satisfies the feedback constraints, if possible.
3. Make N_E equal to $N_{11} * N_{12} * (I_{13} + F_{12,11})$.
4. Find $N_{11} * N_{12}$ by horizontal merging of N_{11} and N_{12}.
5. Derive $I_{13} + F_{12,11}$ from N_E, if possible.
6. Derive $F_{12,11}$ from $I_{13} + F_{12,11}$, if possible.

Feedback Rule Based Networks

<u>Network with Multiple Global Feedback</u>

- at least two sets of nodes embraced by separate feedback with at least two adjacent nodes in each set

N1(F1)	N2(F1)
N1(F2)	N2(F2)

N1(F1)	N1(F2)
N2(F1)	N2(F2)

{N1(F1), N2(F1)}, {N1(F2), N2(F2)} – sets of nodes embraced by separate feedback

Feedback Rule Based Networks

Example 7 (Network Analysis)

$[N_{11}]\ (x_{11}, w_{12,11} \mid z_{11,12}^{1,1}) * [N_{12}]\ (z_{11,12}^{1,1}, w_{13,12} \mid z_{12,13}^{1,1}, v_{12,11}) *$

$[N_{13}]\ (z_{12,13}^{1,1} \mid y_{13}, v_{13,12}),$

$[F_{12,11}]\ (v_{12,11} \mid w_{12,11}), [F_{13,12}]\ (v_{13,12} \mid w_{13,12}) \Rightarrow$

$\{[N_{11}]\ (x_{11}, w_{12,11} \mid z_{11,12}^{1,1}) + [I_{31}]\ (w_{13,12} \mid w_{13,12})\} *$

$[N_{12}]\ (z_{11,12}^{1,1}, w_{13,12} \mid z_{12,13}^{1,1}, v_{12,11}) *$

$\{[N_{13}]\ (z_{12,13}^{1,1} \mid y_{13}, v_{13,12}) + [I_{33}]\ (v_{12,11} \mid v_{12,11})\} *$

Feedback Rule Based Networks

$\{[I_{14}]\ (y_{13} \mid y_{13}) + [F_{13,12}]\ (v_{13,12} \mid w_{13,12}) +$

$[F_{12,11}]\ (v_{12,11} \mid w_{12,11})\} \Rightarrow$

$[(N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33}) * (I_{14} + F_{13,12} + F_{12,11})]$

$(x_{11}, w_{11}, w_{12} \mid y_{12}, w_{12}, w_{11}) \Rightarrow$

$[((N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33}) * (I_{14} + F_{13,12} + F_{12,11}))^{FE}]$

$(x_{11}, x_1^{FE}, x_2^{FE} \mid y_{12}, y_2^{FE}, y_1^{FE}) \Rightarrow$

Feedback Rule Based Networks

$N_E = ((N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33}) * (I_{14} + F_{13,12} + F_{12,11}))^{FE}$

$F_{12,11}$, $F_{13,12}$ – feedback nodes
I_{31}, I_{33}, I_{14} – identity nodes

Algorithm 7.1 (Network Design)
1. Define N_E, N_{11}, N_{12}, N_{13}, I_{14}, I_{31}, I_{33} and $F_{13,12}$.
2. Confirm that N_E satisfies the feedback constraints, if possible.
3. Make N_E equal to
$(N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33}) * (I_{14} + F_{13,12} + F_{12,11})$.
4. Find $N_{11} + I_{31}$ by vertical merging of N_{11} and I_{31}.
5. Find $(N_{11} + I_{31}) * N_{12}$ by horizontal merging of $(N_{11} + I_{31})$ and N_{12}.

Feedback Rule Based Networks

6. Find $N_{13} + I_{33}$ by vertical merging of N_{13} and I_{33}.
7. Find $(N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33})$ by horizontal merging of $(N_{11} + I_{31}) * N_{12}$ and $(N_{13} + I_{33})$.
8. Derive $I_{14} + F_{13,12} + F_{12,11}$ from N_E, if possible.
9. Find $I_{14} + F_{13,12}$ by vertical merging of I_{14} and $F_{13,12}$.
10. Derive $F_{12,11}$ from $I_{14} + F_{13,12} + F_{12,11}$, if possible.

Algorithm 7.2 (Network Design)

1. Define N_E, N_{11}, N_{12}, N_{13}, I_{14}, I_{31}, I_{33} and $F_{11,12}$.
2. Confirm that N_E satisfies the feedback constraints, if possible.
3. Make N_E equal to $(N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33}) * (I_{14} + F_{13,12} + F_{12,11})$.
4. Find $N_{11} + I_{31}$ by vertical merging of N_{11} and I_{31}.

Feedback Rule Based Networks

5. Find $(N_{11} + I_{31}) * N_{12}$ by horizontal merging of $(N_{11} + I_{31})$ and N_{12}.
6. Find $N_{13} + I_{33}$ by vertical merging of N_{13} and I_{33}.
7. Find $(N_{11} + I_{31}) * N_{12} * (N_{13} + I_{33})$ by horizontal merging of $(N_{11} + I_{31}) * N_{12}$ and $(N_{13} + I_{33})$.
8. Derive $I_{14} + F_{13,12} + F_{12,11}$ from N_E, if possible.
9. Derive $F_{13,12}$ from $I_{14} + F_{13,12} + F_{12,11}$, if possible.

Evaluation of Rule Based Networks

Structural Evaluation Metrics

- number of non-identity network nodes
- number of non-identity network connections
- overall number of cells in grid structure
- number of populated cells in grid structure
- average path length from first layer nodes to last layer nodes
- average path depth from first level nodes to last level nodes

Evaluation of Rule Based Networks

<u>Linguistic Evaluation Metrics</u>

- composing hierarchical into standard rule based systems
- decomposing standard into hierarchical rule based systems

Composition Formula

$N_{E,x} = *_{p=1}^{x-1} (N_{1,p} + +_{q=p+1}^{x-1} I_{q,p})$

$N_{E,x}$ – equivalent node for a rule based network with x inputs

Evaluation of Rule Based Networks

Decomposition Algorithm

1. Find $N_{1,1}$ from the first two inputs and the output.
2. If x=2, go to step 9.
3. Set k=3.
4. While $k \leq x$, do steps 5-7.
5. Find $N_{E,k}$ from the first k inputs and the output.
6. Derive $N_{1,k-1}$ from the formula for $N_{E,k}$, if possible.
7. Set k=k+1.
8. Endwhile.
9. End.

$N_{E,k}$ – equivalent node for a rule based network with first k inputs

Evaluation of Rule Based Networks

Functional Evaluation Metrics

- model performance indicators
- applications to case studies

Feasibility Index (FI)

$$FI = \mathrm{sum}_{i=1}^{n} (p_i / n)$$

n – number of non-identity nodes

p_i – number of inputs to the i-th non-identity node

lower FI $\Rightarrow$ better feasibility

Evaluation of Rule Based Networks

Accuracy Index (AI)

$$AI = \text{sum}_{i=1}^{nl} \text{ sum}_{j=1}^{qil} \text{ sum}_{k=1}^{vji} (|y_{ji}^{k} - d_{ji}^{k}| / vij)$$

nl – number of nodes in the last layer
qil – number of outputs from the i-th node in the last layer
vji – number of discrete values for the j-th output from the i-th node in the last layer
y_{ji}^{k}, d_{ji}^{k} – simulated and measured k-th discrete value for the j-th output from the i-th node in the last layer
lower AI $\Rightarrow$ better accuracy

Evaluation of Rule Based Networks

Efficiency Index (EI)

$EI = \text{sum}_{i=1}^{n} (q_i^{IOM} . r_i^{IOM})$

n – number of non-identity nodes
IOM – input output mapping
q_i^{IOM} – number of outputs from the i-th non-identity node with an associated IOM
r_i^{IOM} – number of rules for the i-th non-identity node with an associated IOM
lower EI $\Rightarrow$ better efficiency

Evaluation of Rule Based Networks

Transparency Index (TI)

$$TI = (p + q) / (n + m)$$

p – overall number of inputs
q – overall number of outputs
n – number of non-identity nodes
m – number of non-identity connections
lower TI $\Rightarrow$ better transparency

Evaluation of Rule Based Networks

Case Study 1 (Ore Flotation)

Inputs:
x_1 – copper concentration in ore pulp
x_2 – iron concentration in ore pulp
x_3 – debit of ore pulp

Output:
y – new copper concentration in ore pulp

Evaluation of Rule Based Networks

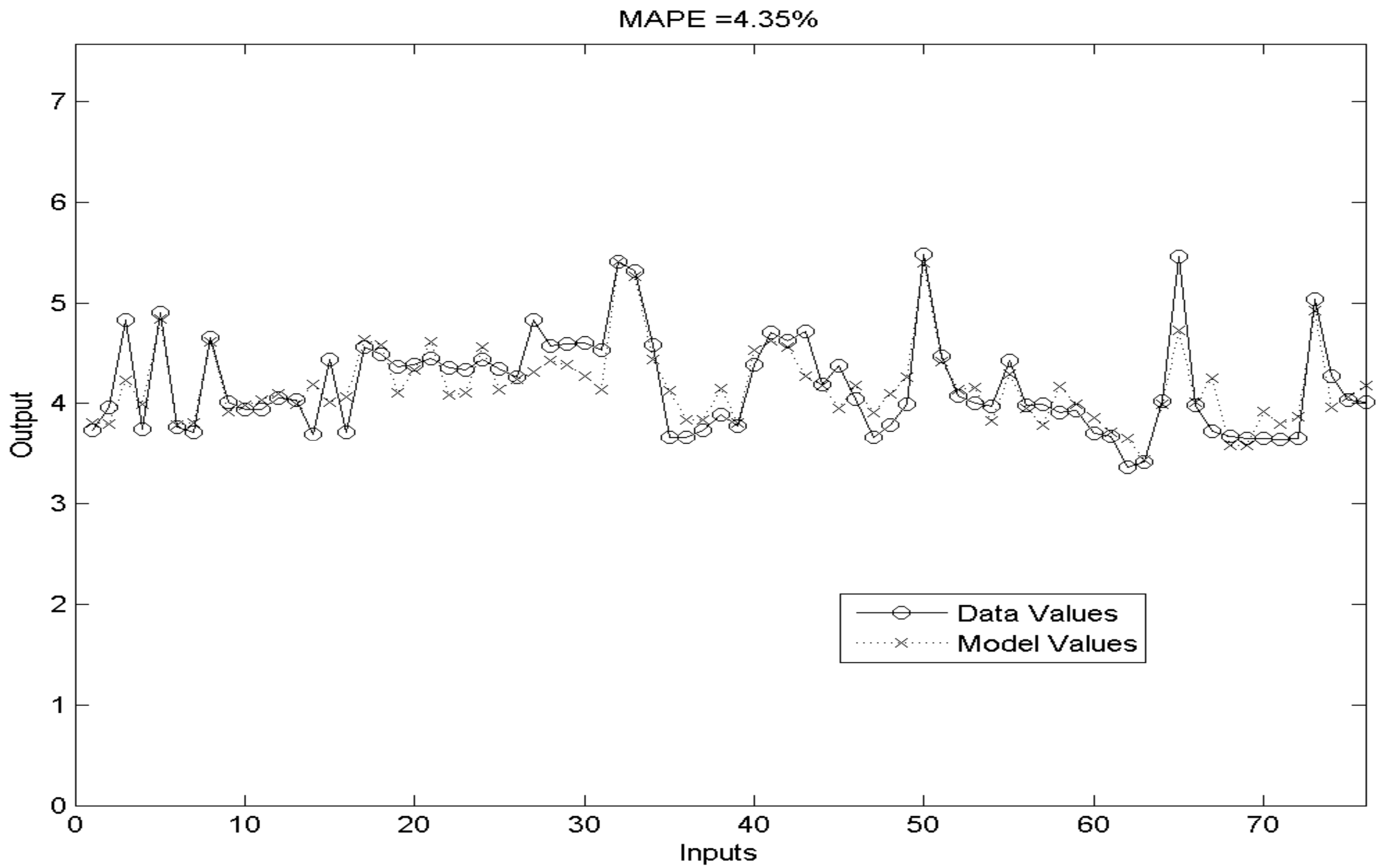


Figure 4: Standard rule based system for case study 1

Evaluation of Rule Based Networks

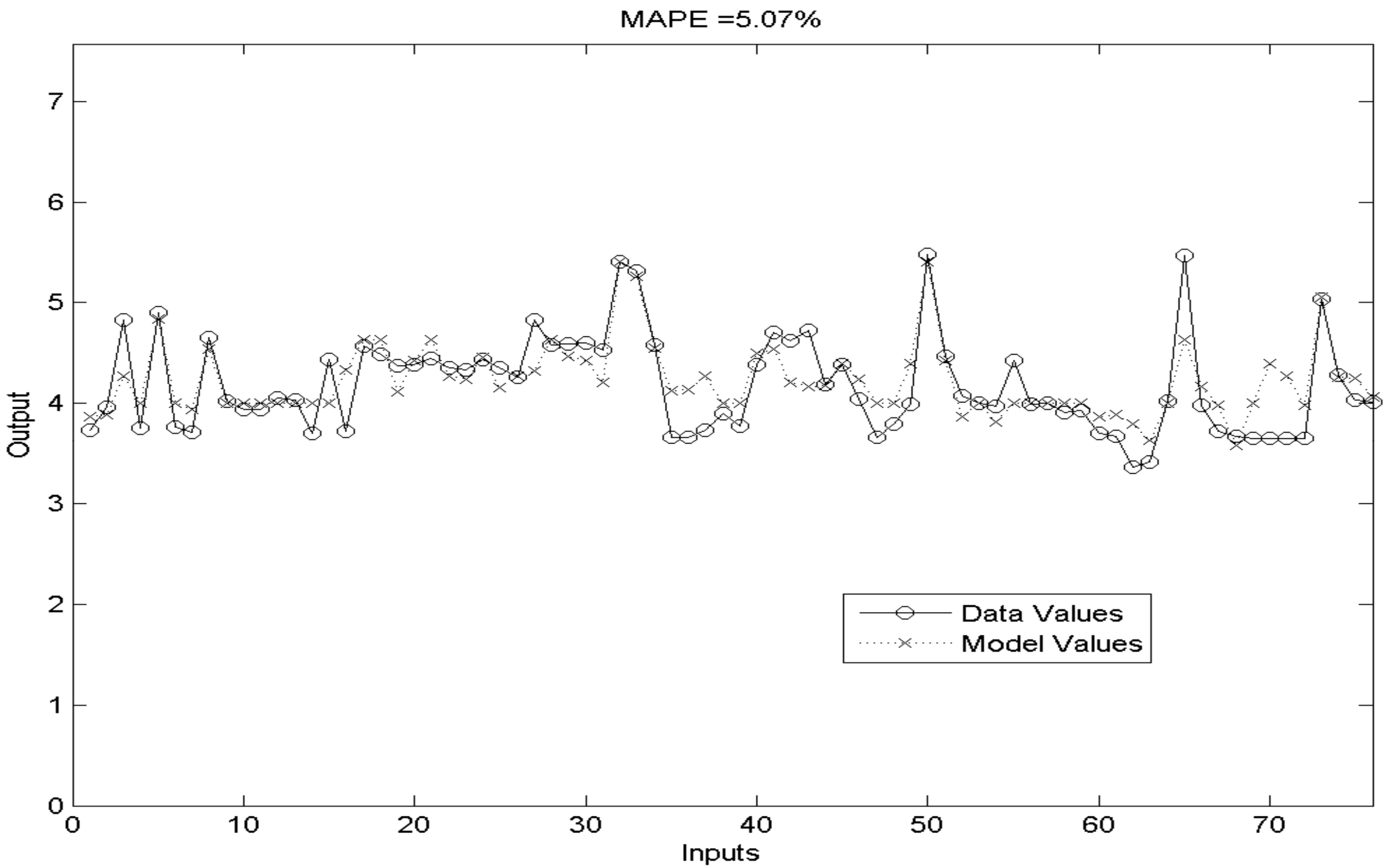


Figure 5: Hierarchical rule based system for case study 1

Evaluation of Rule Based Networks

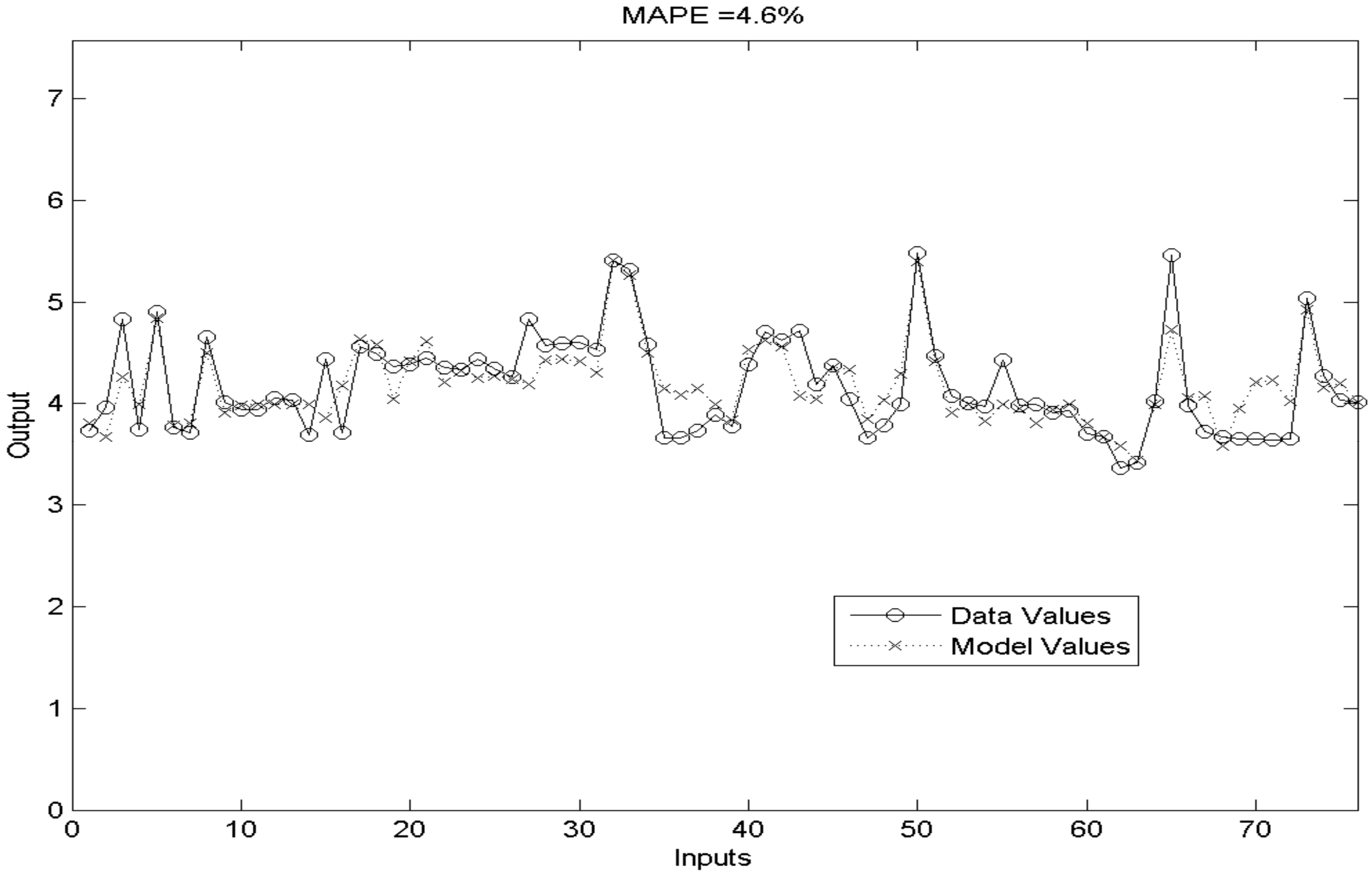


Figure 6: Rule based network for case study 1

Evaluation of Rule Based Networks

Table 1: Models performance for case study 1

Index / Model	Standard rule based system	Hierarchical rule based system	Rule based network
Feasibility	3	2	2
Accuracy	4.35	4.76	4.60
Efficiency	1331	242	1331
Transparency	4	1.33	1.33

FI – RBN superior to SRBS and equivalent to HRBS
AI – RBN inferior to SRBS and superior to HRBS
EI – RBN equivalent to SRBS and inferior to HRBS
TI – RBN superior to SRBS and equivalent to HRBS

Evaluation of Rule Based Networks

Case Study 2 (Retail Pricing)

Inputs:
x_1 – expected selling price for retail product
x_2 – difference between selling price and cost for retail product
x_3 – expected percentage to be sold from retail product

Output:
y – maximum cost for retail product

Evaluation of Rule Based Networks

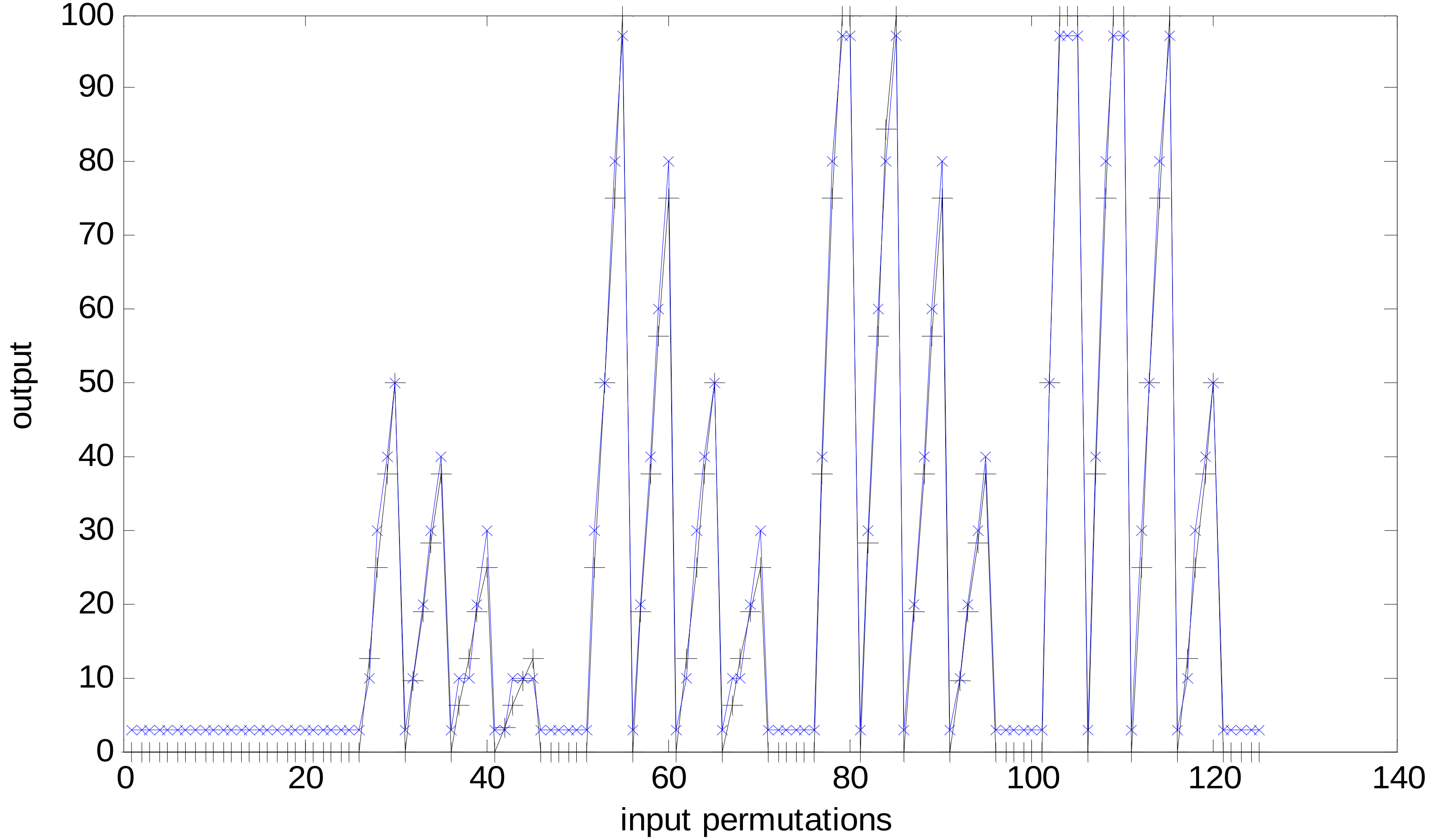


Figure 7: Standard rule based system for case study 2

Evaluation of Rule Based Networks

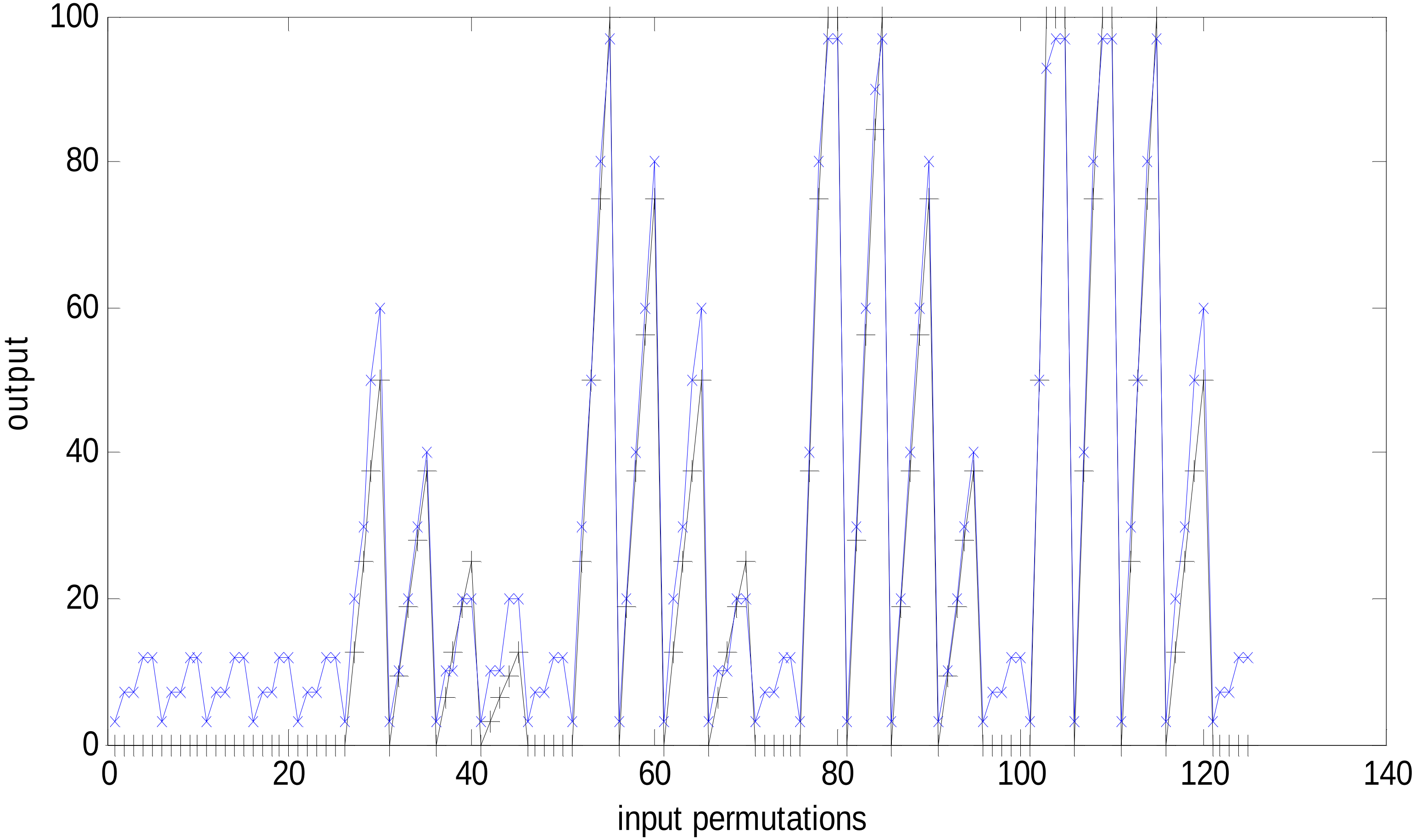


Figure 8: Hierarchical rule based system for case study 2

Evaluation of Rule Based Networks

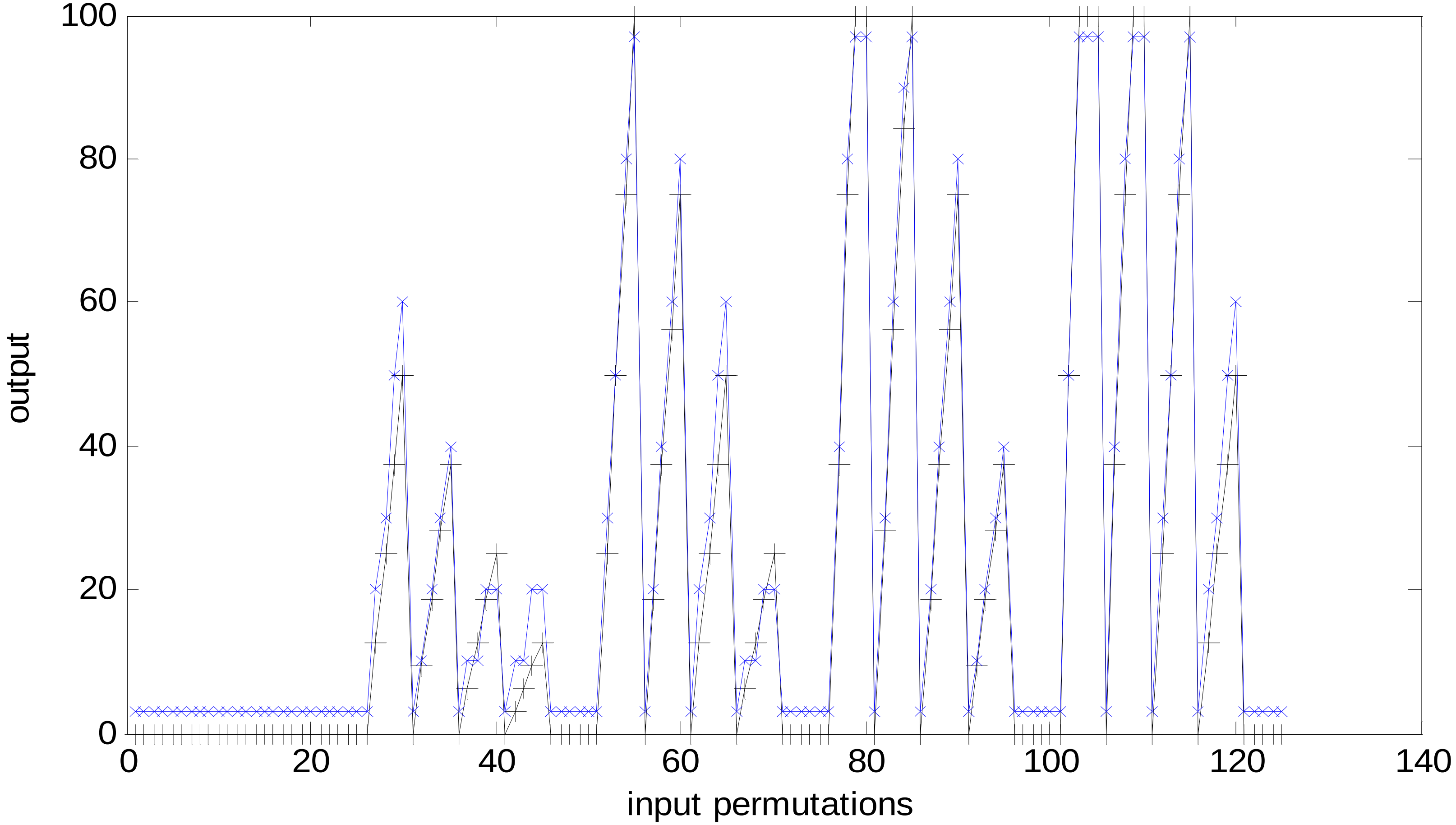


Figure 9: Rule based network for case study 2

Evaluation of Rule Based Networks

Table 2: Models performance for case study 2

Index / Model	Standard rule based system	Hierarchical based based system	Rule based network
Feasibility	3	2	2
Accuracy	2.86	5.57	3.64
Efficiency	125	80	125
Transparency	4	1.33	1.33

FI – RBN superior to SRBS and equivalent to HRBS
AI – RBN inferior to SRBS and superior to HRBS
EI – RBN equivalent to SRBS and inferior to HRBS
TI – RBN superior to SRBS and equivalent to HRBS

Evaluation of Rule Based Networks

Composition of Hierarchical into Standard Rule Based System

- full preservation of model feasibility
- maximal improvement of model accuracy
- fixed loss of model efficiency
- full preservation of model transparency

Decomposition of Standard into Hierarchical Rule Based System

- no change of model feasibility
- minimal loss of model accuracy
- fixed improvement of model efficiency
- no change of model transparency

Rule Based Network Toolbox

Details

- developed by Erasmus and project students
- implemented in the Matlab environment
- downloadable from the Mathworks web site
- visible from the Springer web site

Structure

- Matlab files for basic operations on nodes
- Matlab files for advanced operations on nodes
- Matlab files for auxiliary operations
- Word files with illustration examples

Conclusion

Theoretical Significance of Rule Based Networks

- novel application of discrete mathematics and control theory
- detailed validation by test examples and case studies

Methodological Impact of Rule Based Networks

- extension of standard and hierarchical rule based systems
- bridge between standard and hierarchical rule based systems
- novel framework for different types of rule based systems

Application Areas of Rule Based Networks

- modelling and simulation in the mining industry
- modelling and simulation in the retail industry

Conclusion

<u>Other Applications of Rule Based Networks</u>

- finance (mortgage evaluation)
- healthcare (patient management)
- transport (commuter modelling)
- robotics (path planning)
- construction (crane control)
- football (score prediction)
- hospitality (rice cooking)
- navigation (boat sailing)
- environment (weather forecasting)
- business (risk analysis)

References

A.Gegov, Fuzzy Networks for Complex Systems:
A Modular Rule Base Approach, Springer, Berlin, 2010