Methods for System Performance Improvement of Shadowed Fading Channels

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- Wireless channels are simultaneously affected by short-term fading and long-term fading (shadowing)
- Shadowing is the result of the topographical elements and other structures in the transmission path such as trees, tall buildings...

- Short-term fading (multipath fading) is propagation phenomenon caused by atmospheric ducting, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings

- Various statistical models explain the nature of fading and several distributions describe fading statistics: Rayleigh, Rice, Nakagami-*m*, Hoyt, Weibull, α-μ, k- μ, …

- Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices
- Rayleigh fading models assume that the magnitude of a signal that has passed through such a transmission medium (also called a communication channel will vary randomly, or fade, according to a Rayleigh distribution

- In Rayleigh distribution — the radial component of the sum of two uncorrelated Gaussian random variables
- Rayleigh fading is viewed as a reasonable model for tropospheric and ionosperic signal propagation as well as the effect of heavily built-up urban environments on radio signals

- Rayleigh fading is most applicable when there is no dominant propagation along a line of sight between the transmitter and receiver
- If there is a dominant line of sight, Rician fading may be more applicable

- Nakagami-*m* distribution is preferred because of wide range of its applicability and mathematical tractability
- It can be reduced to Rayleigh distribution for appropriate value of parameter *m*

- The main problem is to find such distribution which adequately fit to measured data
- One of the distributions that show good fit with experimental data is Weibull distribution

- Another distribution, which has recently received increased attention in modelling fading channels is the Hoyt or Nakagami-*q* distribution

- Several studies have shown that the Hoyt fading model provides a very accurate fit to experimental channel measurements in a various communication applications, like mobile satellite propagation channels

- Hoyt fading spans the range of the fading figure from the one-sided Gaussian to the Rayleigh distribution
- Similarly, the Hoyt distribution can be considered as an accurate fading model for satellite links with strong ionospheric scintillation

- To describe the fading models, besides the Weibull, there is a more generalized α-μ distribution
- It is valid for the non-linearity of the propagation medium as well as for the multipath clustering

- The α-μ distribution is a general fading distribution that can be used to represent the small-scale variation of the fading signal in a non-line-of-sight fading condition
- This fading model providing a very good fit to measured data over a wide range of fading conditions

- This distribution has two physical parameters, α and μ
- The parameter α is related to the non-linearity of the environment, whereas the parameter μ is associated to the number of multipath clusters

- The k-μ distribution is also general distribution
- κ-μ - fading has been chosen because it includes, as special cases, Nakagami-*m* and Nakagami-*n* (Rice) fading, and their entire special cases as well (e.g., Rayleigh and one-sided Gaussian fading)

- Also, the sum of κ-μ squares is κ-μ square as well (with different parameters)
- This is an ideal choice for MRC combining analysis

- Concerning this, this model for *κ-μ* distribution and closed form expressions for the outage probability, BEP and SEP at the MRC output could be derived for a broad class of modulation types

- By setting $k=0$, k-μ distribution reduces to Nakagami-*m* distribution
- For $\mu=1$ and $k=0$, from k-μ distribution Rayleigh distribution can be derived
- Rice distribution is obtained from k-μ distribution for $\mu=1$ as special case

- Diversity technique is one of the most used methods for minimizing fading effect and increasing the communication reliability without enlarging either transmitting power or channel's bandwidth

- Diversity techniques combine the multiple received signals in reception device, on different ways

- There are several types of diversity combining techniques:
- \- Maximum ratio combining (MRC) and
- \- Equal gain combining (EGC) techniques

- They require more information about channel: fading amplitude, phase and delay

- The implementation of these diversity techniques is quite complex and expensive since they require a separate receiver for each branch

- Selection combining (SC) diversity technique is simpler for implementation because the SC systems process only one of the diversity branches

- If the noise power is equally distributed over branches, SC receiver selects the branch with the highest signal-to-noise ratio (SNR) and that is the branch with the strongest signal

- In fading environments, when the level of noise is sufficiently low compared with the level of co-channel interference (CCI) SC combiner processes the branch with the highest signal-to-interference ratio (SIR-based selection diversity)

- Swith andb Stay combining (SSC) receiver selects one antenna until its quality falls below a predetermined threshold
- After this, receiver switches to second antenna whereas the signal from this antenna is below or above the threshold

- In communication systems where antennas are sufficiently apart, it is considered there is no correlation between transmitted signals, as well as between interferences at the reception

- However, it can not be always done in practice because there is insufficient antenna spacing when diversity is applied in small devices.
- Performance of diversity systems in which there is a correlation between transmitted signals and between interferers have to be considered also

- Fading statistic does not necessary have to be the same in each branch
- **For example**, PDF can be the same, but with different parameters (Nakagami-*m* fading in *i*-th and *j*-th branches, with m_i do not equal to m_j), or PDFs in different branches are different (Nakagami-*m* fading in *i*-th branch, and Rice fading in *j*-th branch)

I part

Macrodiversity and N- Branches Microdiversity Reception

- An efficient method for reducing short-term fading effect at micro-level (single base station) with using multiple receiver antennas is called space diversity

- Upgrading transmission reliability without increasing transmission power and bandwidth while increasing channel capacity is the main goal of space diversity techniques

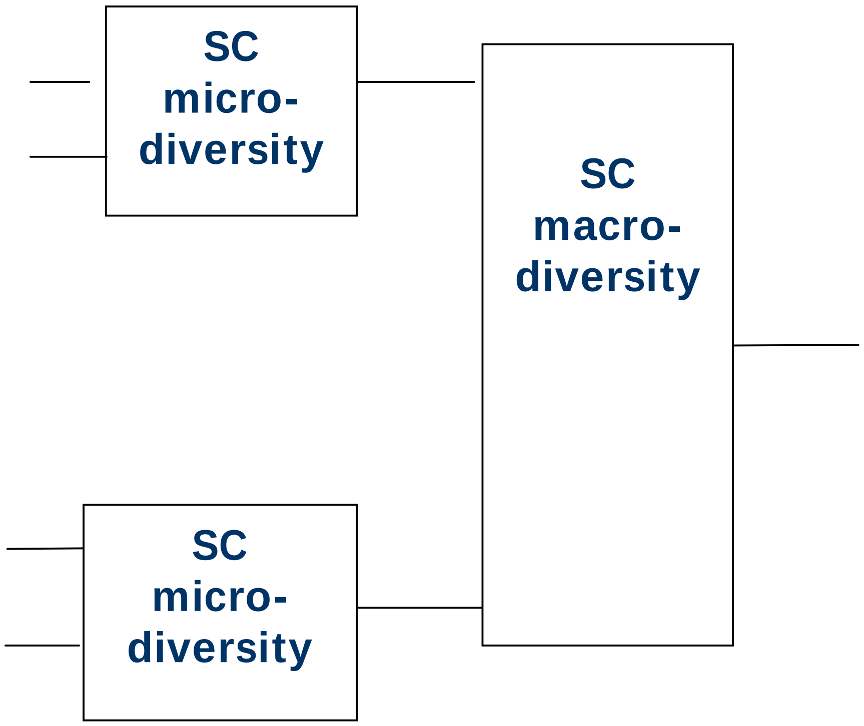
- While short-term fading is mitigated through the use of diversity techniques typically at the single base station (micro-diversity), the use of such micro-diversity approaches alone will not be sufficient to mitigate the overall channel degradation when shadowing is also concurrently present

- Since they coexist in wireless systems, short- and long-term fading conditions must be simultaneously taken into account
- Macro-diversity reception is used to alleviate the effects of shadowing, where multiple signals are received at widely located base stations

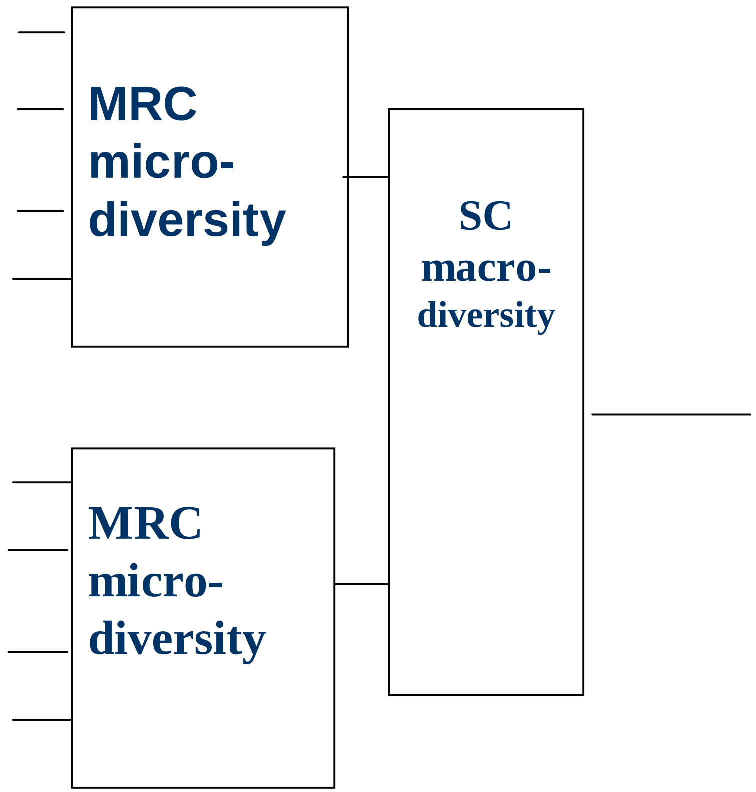
- At the macro-level, selection combining (SC) is used as basically fast response handoff mechanism that instantaneously or, with minimal delay, chooses the best base station to serve mobile based on the signal power received

- In base stations, at the micro-level, selection combining (SC) or maximal ratio combining (MRC) are used
- The system models are presented:

System model 1

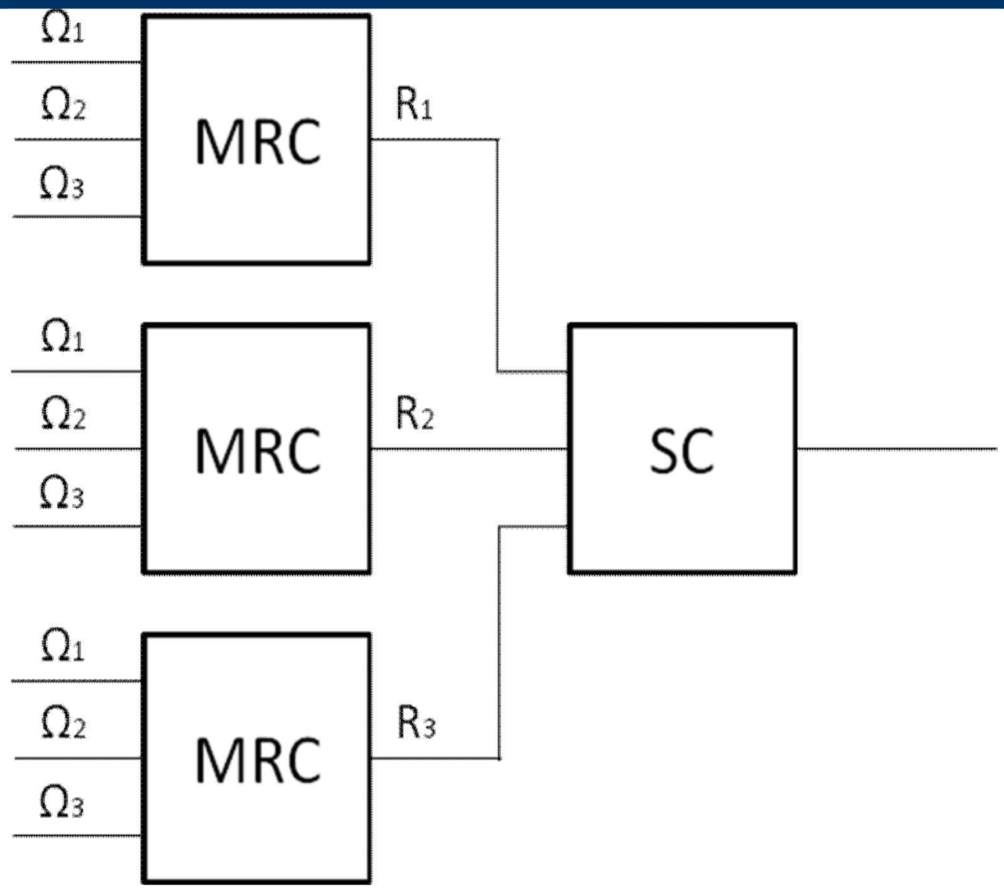


System model 2



- Most of the papers are treating diversity combining have examined only dual-branch combining because of the inability to obtain closed-form expressions for evaluated parameters of diversity system

System model 3



- The wireless communication system following microdiversity to mitigate the effects of short-term fading and macrodiversity processing to reduce shadowing effects

- N-branch maximal-ratio combining (MRC) or selection combining (SC) is implemented at the micro level (single base station)
- Selection combining (SC) with two (dual diversity) or more base stations is implemented at the macro level

- Model assumes a Rice, Rayleigh, Nakagami-*m* or other density function for the envelope of the received signal and a log-normal or gamma distribution to model the average power to account for shadowing

- Log-normal distribution does not lead to the closed form expressions in combined analisis with one of distributions for fast fading
- Because of that Gamma fading is using latterly for shadowing

- Analytical expressions for the probability density function (PDF), cumulative distribution function (CDF) and moments of signal after micro- and macro-diversity processing are derived

- These expressions are used to study important system performance criteria such as the outage probability (OP), average bit error probability (ABEP), average output signal value and amount of fading (AoF)

- Then, various numerical results are graphically presented to illustrate the proposed mathematical analysis and to show the effects of various system parameters to the system performance, as well as enhancement due to use of the combination of micro- and macrodiversity

- However, in certain applications, such as in adaptive transmission, the outage probability and Average Bit Error Probability do not provide enough information for the overall system design and configuration

- In that case, system’s second-order statistics should be obtained to reflect the correlation properties of the fading channels and to provide a dynamic representation of the system’s performance

- The level crossing rate (LCR) and average fade duration (AFD) represent the system’s second-order statistics
- They can be used as important performance measures for a proper selection of the adaptive symbol rates, interleaver depth, packet length and time slot duration

The examples

- Second order statistics of selection macro-diversity system operating over Gamma shadowed Ricean fading channels

- The expression for the pdf of the outputs of MRC micro-diversity systems is

$$p(z_i/\Omega_i) = \frac{L_i(1+K_i)^{\frac{L_i+1}{2}}}{K_i^{\frac{L_i-1}{2}} \exp\left(L_i K_i\right)(L_i\Omega_i)} z_i^{\frac{L_i-1}{2}} \exp\left(-\frac{(1+K_i)z_i}{\Omega_i}\right)$$

$$I_{L_i-1}\left(2L_i\sqrt{\frac{K_i(1+K_i)z_i}{L_i\Omega_i}}\right)$$

- $I_r(x)$ denotes the r-th order modified Bessel function of first kind
- K_i denoting the Ricean factor, defined as the ratio of power in the specular and scattered components of each micro-diversity system
- L_i denotes the number of channels at each micro-level

- The outputs of a MRC system and their derivatives are

$$z_i^2 = \sum_{k=1}^{L_i} z_{ik}^2 \qquad \dot{z}_i = \sum_{k=1}^{L_i} \frac{z_{ik}}{z_i} \dot{z}_{ik}$$

- $\dot{z}_i$ is a Gaussian random variable with a zero mean

$$p(\hat{z}_i) = \frac{1}{\sqrt{2\pi}\,\hat{\sigma}_{z_i}} \exp\left(-\frac{\hat{z}_i^{\,2}}{2\hat{\sigma}_{z_i}^{\,2}} \right)$$

- The variance is given with

$$\hat{\sigma}_{z_i}^{\,2} = \frac{\sum_{k=1}^{N_i} z_{ik}^2 \hat{\sigma}_{z_{ik}}^{\,2}}{\sum_{k=1}^{N_i} z_{ik}^2}$$

- Conditioned on Ωj, the joint PDF can be calculated as:

$$p_{z_i,\tilde{z}_i|\Omega_i}(z_i,\tilde{z}_i \mid \Omega_i) = \frac{L_i(1+K_i)^{\left(\frac{L_i+1}{2}\right)}}{K_i^{\frac{(L_i-1)}{2}} \exp\left(L_i K_i\right)(L_i\Omega_i)} z_i^{\frac{L_i-1}{2}}$$

$$\times \exp\left(-\frac{(K_i+1)z_i}{\Omega_i}\right) I_{L_i-1}\left(2L_i\sqrt{\frac{K_i(1+K_i)z_i}{L_i\Omega_i}}\right)$$

$$\times \frac{1}{\sqrt{2\pi}\tilde{\sigma}_{z_i}} \exp\left(-\frac{\tilde{z}_i^{\,2}}{2\tilde{\sigma}_{z_i}^{\,2}}\right); \quad i = 1,2$$

- The macro-diversity system is of SC type
- the selection is based on the micro-combiners output signal power values
- At the macro-level, this type of selection is used as handoff mechanism, that chooses the best base station to serve mobile, based on the signal power received

- The unconditional joint probability density of the Z and $\breve{Z}$ is obtained by averaging over the joint pdf p $_{\Omega_1,\Omega_2}(\Omega_1, \Omega_2)$ as

$$p_{z,\breve{z}}(z,\breve{z}) = \int_0^{\infty} d\Omega_1 \int_0^{\Omega_1} p_{z_1,\breve{z}_1|\Omega_1}(z,\breve{z} \mid \Omega_1) \times p_{\Omega_1,\Omega_2}(\Omega_1,\Omega_2)\, d\Omega_2 +$$

$$+\int_0^{\infty} d\Omega_2 \int_0^{\Omega_2} p_{z_2,\breve{z}_2|\Omega_2}(z,\breve{z} \mid \Omega_2)\, p_{\Omega_1,\Omega_2}(\Omega_1,\Omega_2)\, d\Omega_1$$

- The cumulative distribution function (CDF) at the macro-diversity output is

$$F_z(z) = \int_0^\infty d\Omega_1 \int_0^{\Omega_1} F_{z_1|\Omega_1}(z \mid \Omega_1) \times p_{\Omega_1,\Omega_2}(\Omega_1, \Omega_2) d\Omega_2$$

$$+ \int_0^\infty d\Omega_2 \int_0^{\Omega_2} F_{z_2|\Omega_2}(z \mid \Omega_2) \times p_{\Omega_1,\Omega_2}(\Omega_1, \Omega_2) d\Omega_1$$

- If base stations at the macrodiversity level are widely located, due to sufficient spacing between antennas, signal powers at the outputs of the base stations are modelled as statistically independent

- The long-term fading is as described with Gamma distributions:

$$p_{\Omega_1,\Omega_2}(\Omega_1,\Omega_2) = p_{\Omega_1}(\Omega_1)\times p_{\Omega_2}(\Omega_2) =$$

$$= \frac{1}{\Gamma(c_1)}\frac{\Omega_1^{c_1-1}}{\Omega_{01}^{c_1}}\exp\left(-\frac{\Omega_1}{\Omega_{01}}\right)\frac{1}{\Gamma(c_2)}\frac{\Omega_2^{c_2-1}}{\Omega_{02}^{c_2}}\exp\left(-\frac{\Omega_2}{\Omega_{02}}\right)$$

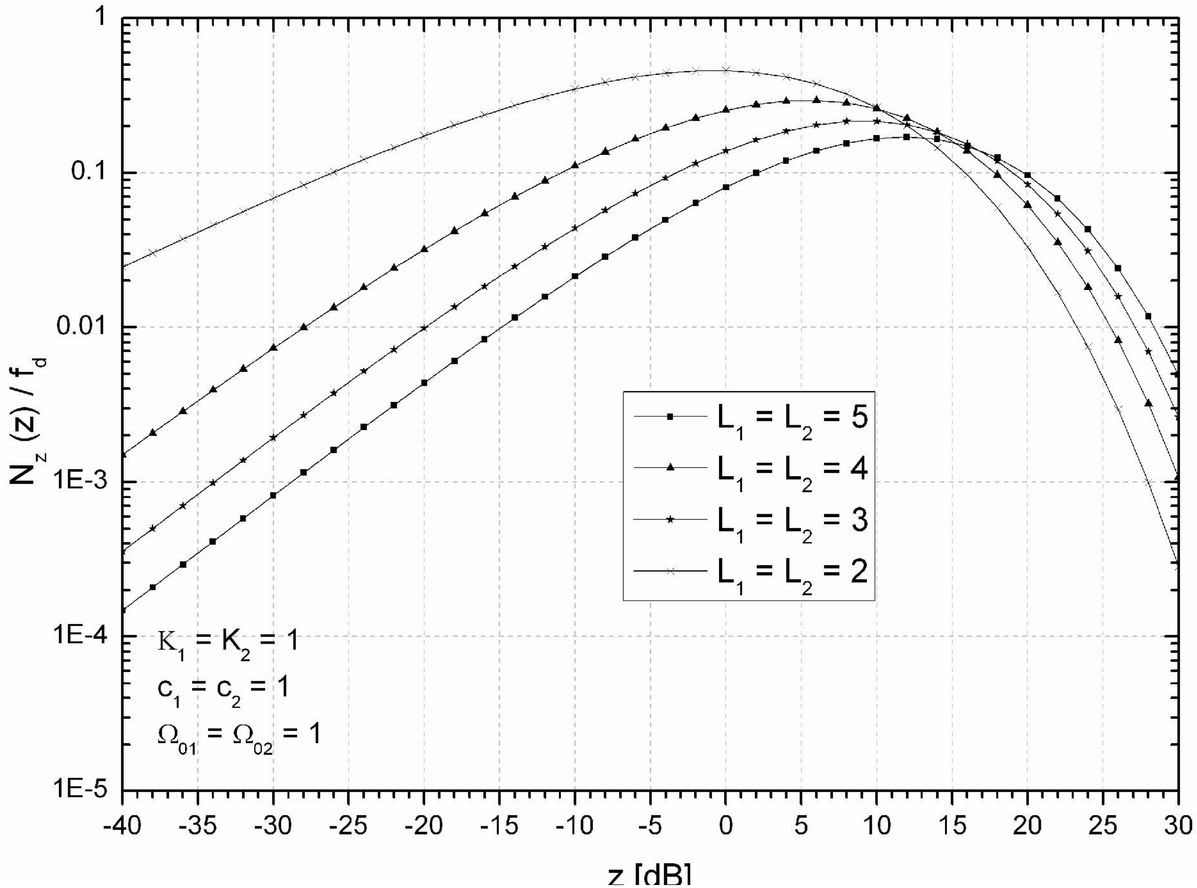
- The level crossing rate (LCR) at the envelope *z* is defined as the rate at which fading signal envelope crosses level *z* in positive or negative direction and is mathematically defined by formula:

$$N_z(z) = \int_0^{\infty} \dot{z}\, p_{z,\dot{z}}(z, \dot{z})\, d\dot{z}$$

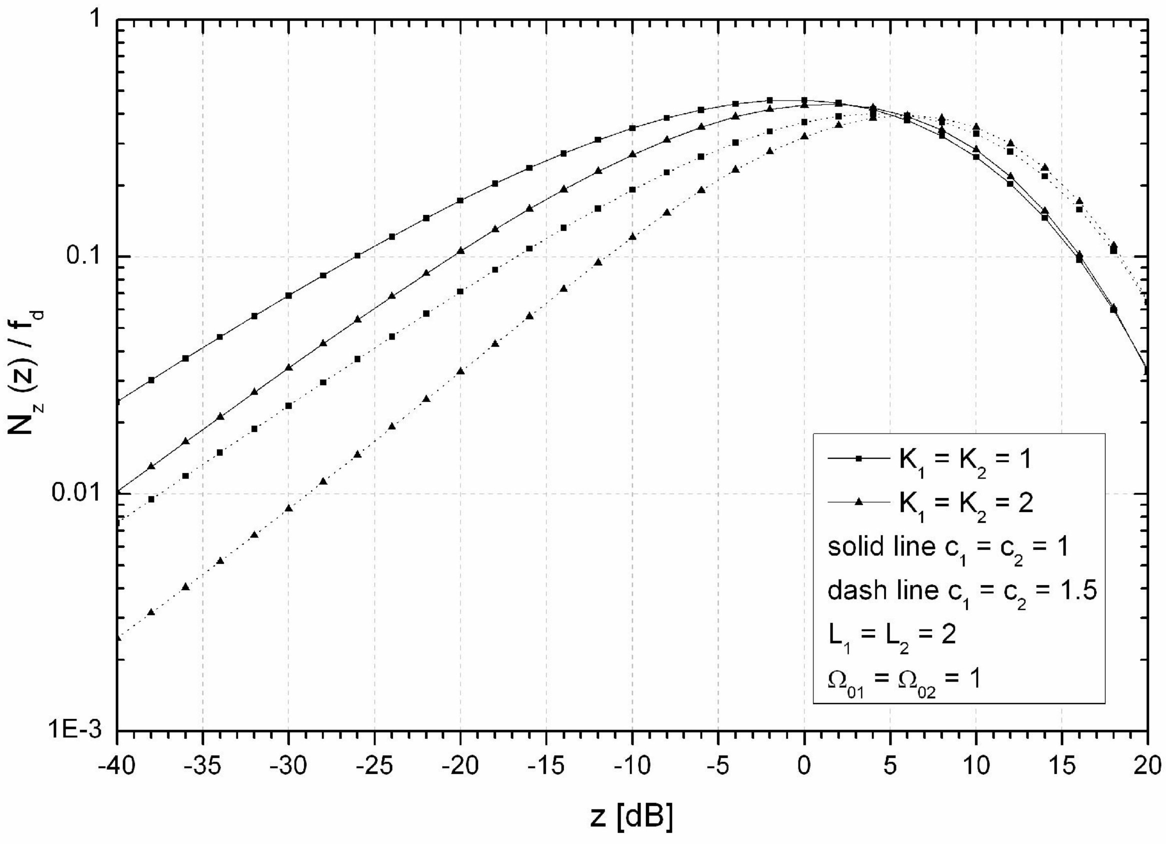
- The average fade duration (AFD) is defined as the average time over which the signal envelope ratio remains below the specified level after crossing that level in a downward direction, and is determined as:

$$T_z(z) = \frac{F_z(z \le Z)}{N_z(z)}$$

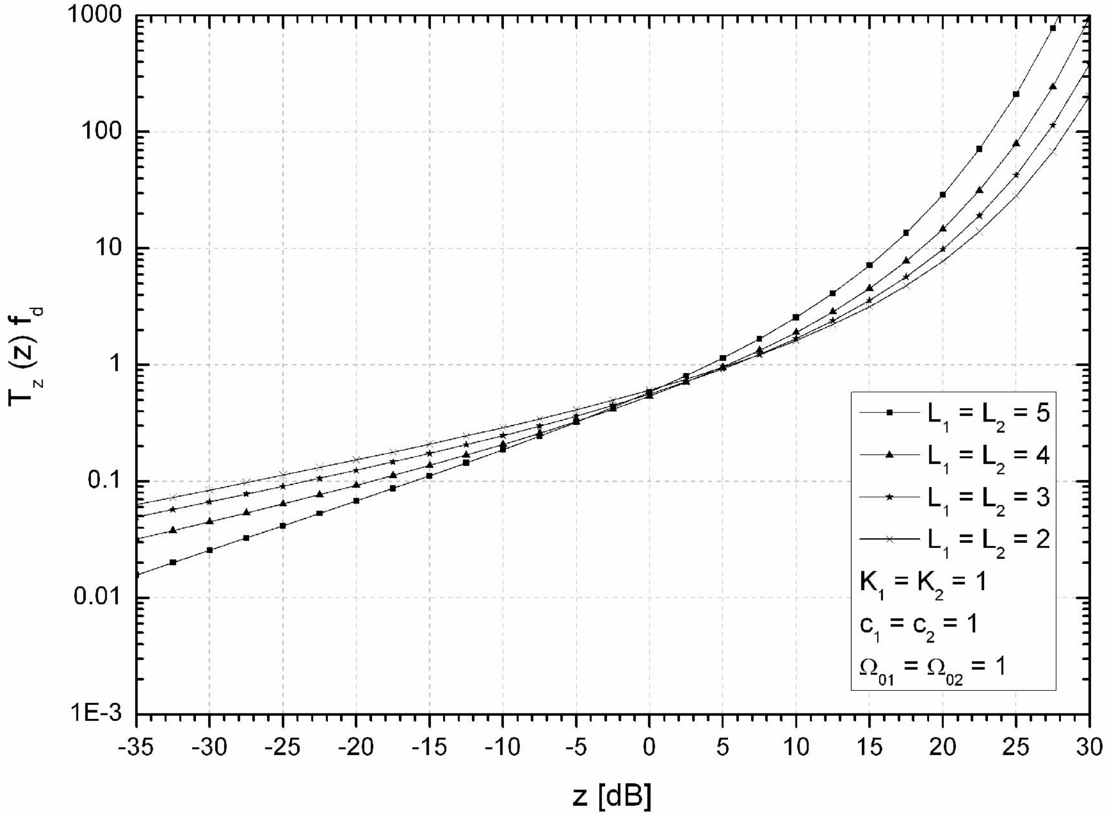
Normalized average LCR of macrodiversity structure for various values of diversity order



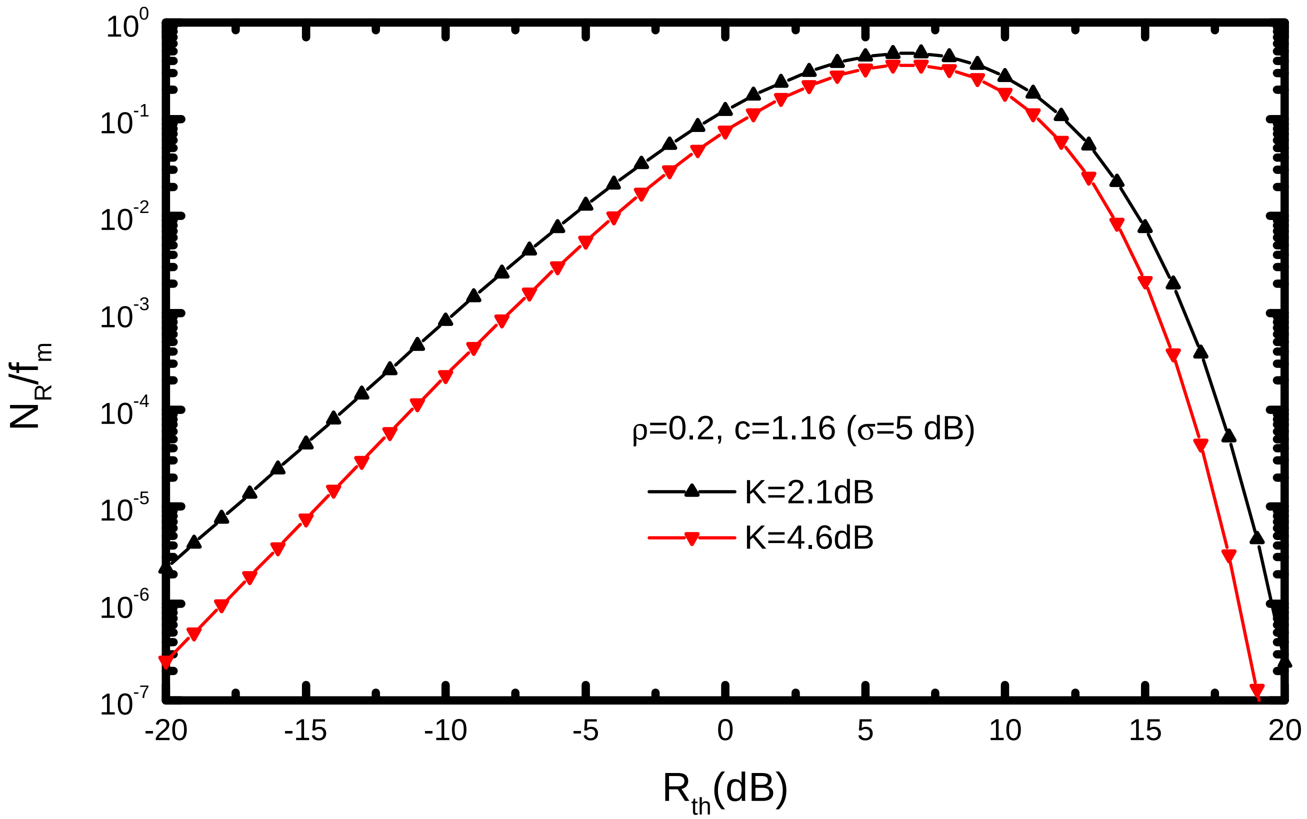
Normalized average LCR of macro diversity structure for various values of shadowing severity levels and Ricean *K* factor



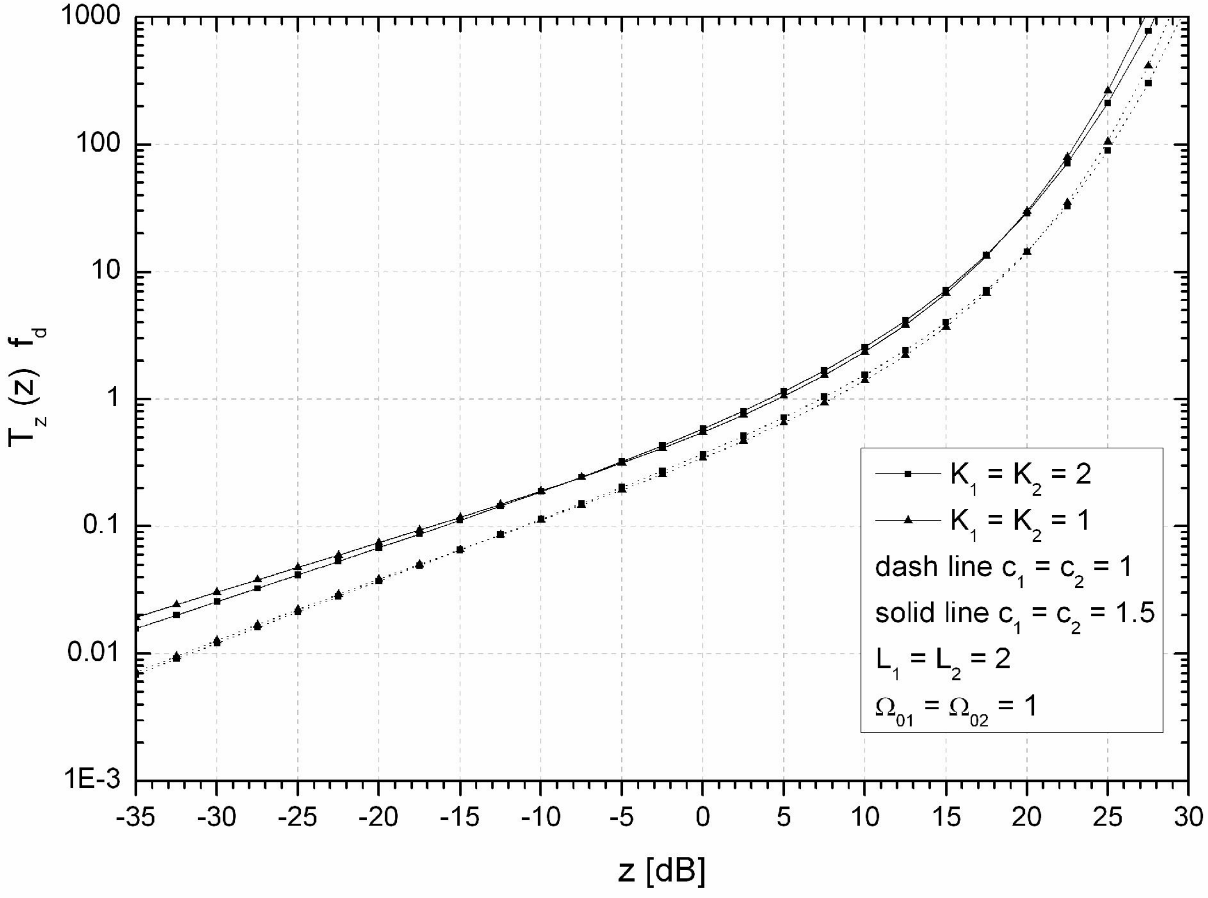
Normalized average AFD of macrodiversity structure for various values of diversity order



Normalized average LCR for different values of fading severity and number of diversity branches at micro level and three branches macrodiversity

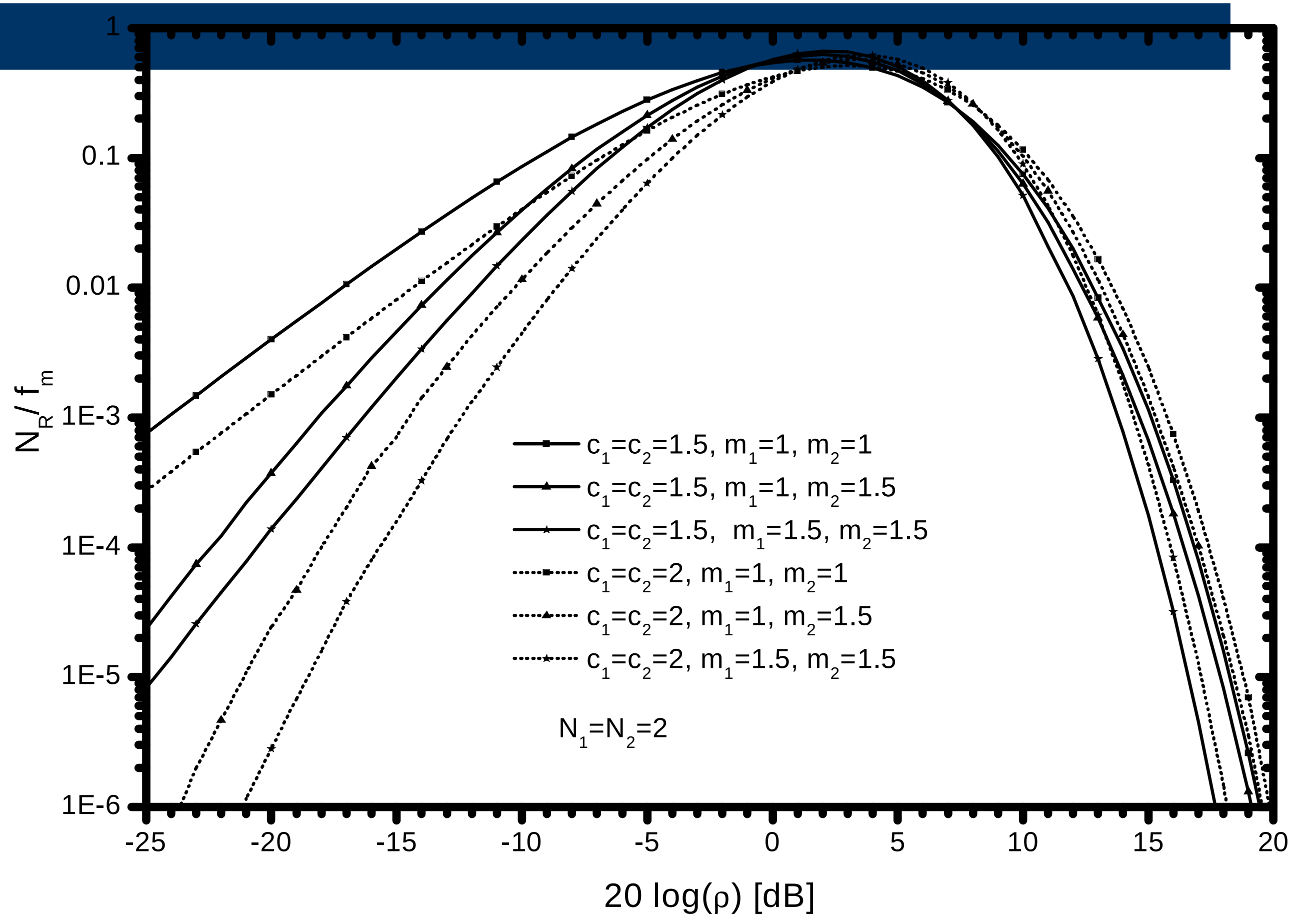


Normalized average AFD of macro diversity structure for various values of shadowing severity levels and Ricean *K* factor

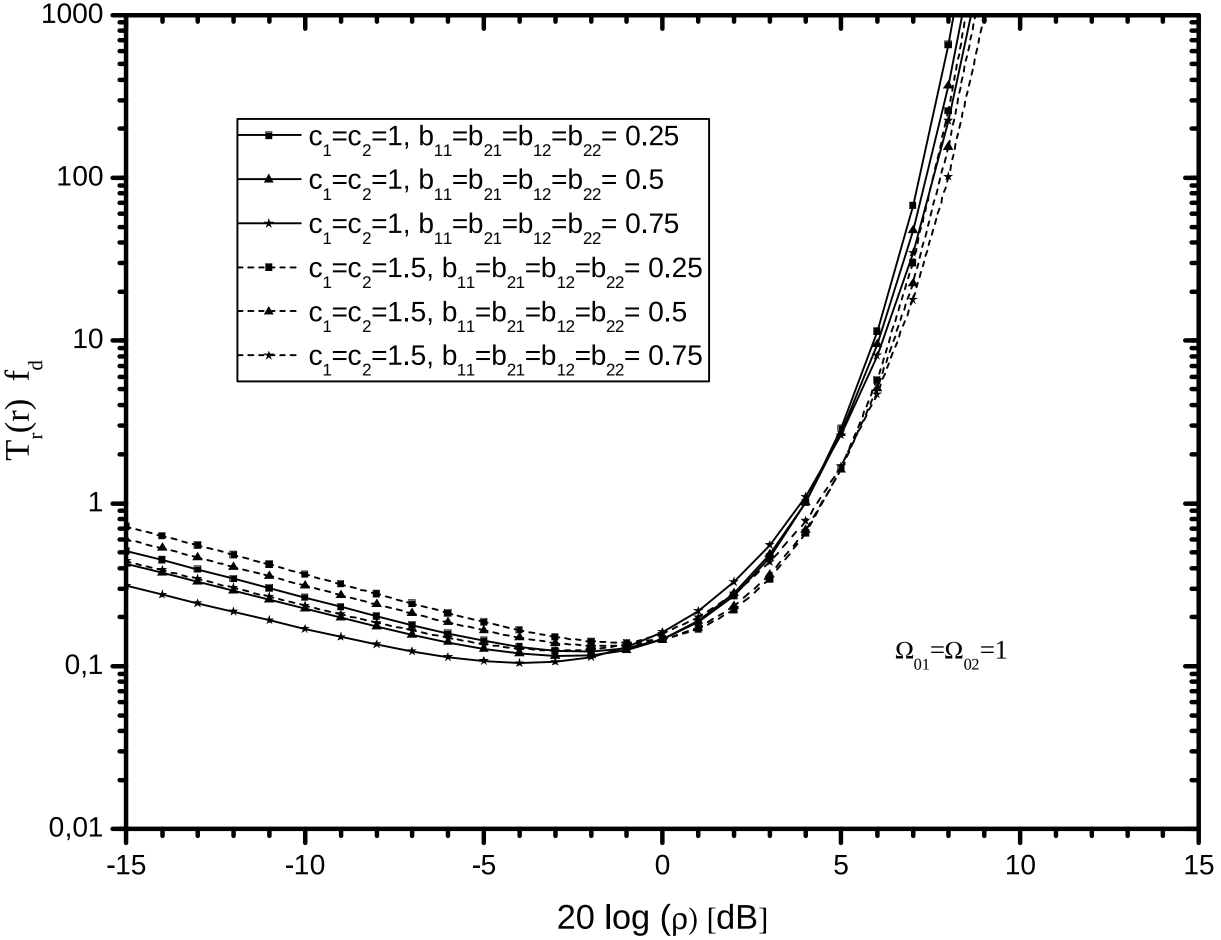


- Some numerical results of the system's Level crossing rate (LCR) and Average Fading Duration (AFD) are presented, in order to examine the influence of various parameters such as shadowing and fading severity and number of the diversity branches at the micro-combiners on concerned quantities

Normalized average LCR of macrodiversity structure versus normalized signal level for various values of Nakagami-*m* fading parameters



Normalized AFD as a function of normalized signal level for various values of system's parameters in the presence of Hoyt Fading



- The average LCR and AFD were presented graphically in order to illustrate the effects of

-number of diversity branches,

-severity of fading and shadowing and

-correlation between base stations

on the system performance

Obtained results can be used for the system parameter optimization in different propagation conditions

II part

Complex combiners:

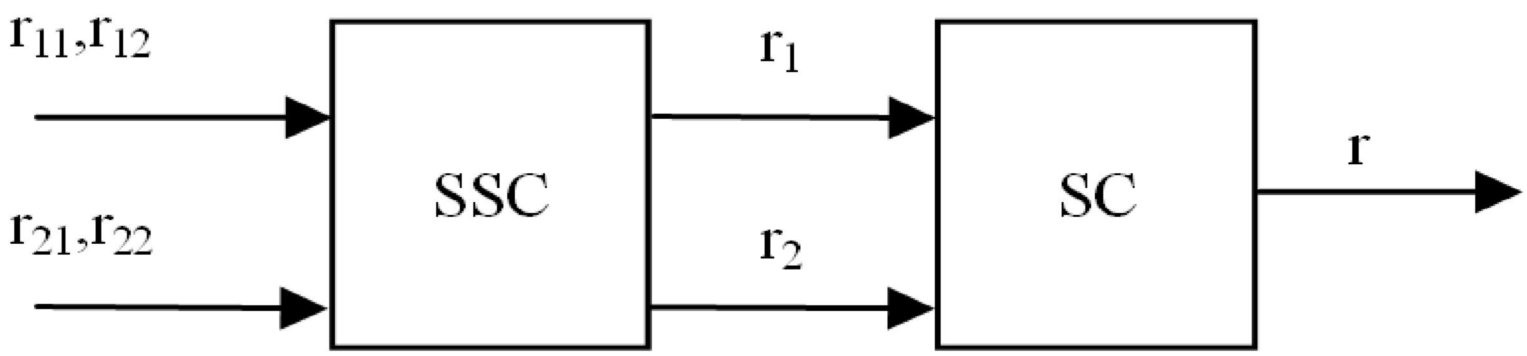
SSC/MRC and

SSC/SC

Characteristics of the SSC Combiner

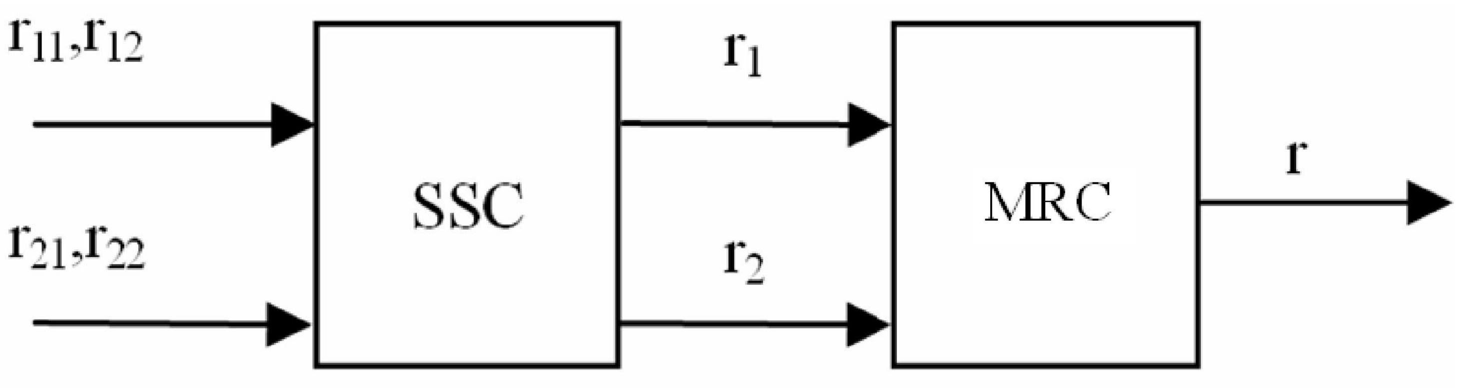
- Model of the system at one time instant
- Model of the system at two time instants

- Joint PDF for SSC Combiner Output Signal at Two Time Instants in Fading Channel
- Probability density function of signal derivatives at the output of SSC combiner at two time instants



System model for complex dual SSC/SC combiner

System model



System model for complex dual SSC/MRC combiner

Introduction

- The complex Switch and Stay Combining/ Selection Combining or Maximal Ratio Combining (SSC/SC or SSC/MRC) combiner are considered

Introduction

- All SSC, SC and MRC combiners are dual-branches
- Up to now, composite combiner at two time instants, has not been considered

System model

- At the inputs of the first part of complex combiner the signals are r_{11} and r_{21} at first time moment
- They are r_{12} and r_{22} at second time moment
- The first index represents the branch ordinal number and the other one signs the time instant observed

System model

- The output signals from SSC part of complex combiner are r_1 and r_2
- The SSC combiner output signals r_1 and r_2 are the inputs for the MRC combiner
- Signals at antennas in branches could be independent or correlated

- Different fading distributions are determined here:
- - Nakagami-m
- - Rayleigh,
- - Rice,...
- - Weibull,
- - Hoyt,
- - Gamma,...

PDF of the combiner output signal at two time instants

for $r_1 < r_T, r_2 < r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11} r_{22} r_{21} r_{12}}(r_{11}, r_{22}, r_1, r_2) + P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21} r_{12} r_{11} r_{22}}(r_{21}, r_{12}, r_1, r_2)$$

for $r_1 \geq r_T,\ r_2 < r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \cdot \int_0^{r_T} dr_{12}\, p_{r_{12} r_{11} r_{22}}(r_{12}, r_1, r_2) + P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11} r_{22} r_{21} r_{12}}(r_{11}, r_{22}, r_1, r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{22}\, p_{r_{22} r_{21} r_{12}}(r_{22}, r_1, r_2) + P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21} r_{12} r_{11} r_{22}}(r_{21}, r_{12}, r_1, r_2)$$

for $r_1 < r_T,\ r_2 \geq r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \cdot \int_0^{r_T} dr_{11}\, p_{r_{11} r_{21} r_{22}}(r_{11}, r_1, r_2) + P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11} r_{22} r_{21} r_{12}}(r_{11}, r_{22}, r_1, r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21}\, p_{r_{21} r_{11} r_{12}}(r_{21}, r_1, r_2) + P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21} r_{12} r_{11} r_{22}}(r_{21}, r_{12}, r_1, r_2)$$

PDF of the combiner output signal at two time instants

for $r_1 \ge r_T, r_2 \ge r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \cdot p_{r_{11} r_{12}}(r_1, r_2) + P_1 \cdot \int_0^{r_T} dr_{12}\, p_{r_{12} r_{11} r_{22}}(r_{12}, r_1, r_2) +$$

$$+ P_1 \cdot \int_0^{r_T} dr_{11}\, p_{r_{11} r_{21} r_{22}}(r_{11}, r_1, r_2) + P_1 \cdot \int_0^{r_T} dr_{11} \int_0^{r_T} dr_{22}\, p_{r_{11} r_{22} r_{21} r_{12}}(r_{11}, r_{22}, r_1, r_2) +$$

$$+ P_2 \cdot p_{r_{21} r_{22}}(r_1, r_2) + P_2 \cdot \int_0^{r_T} dr_{22}\, p_{r_{22} r_{21} r_{12}}(r_{22}, r_1, r_2) +$$

$$+ P_2 \cdot \int_0^{r_T} dr_{21}\, p_{r_{21} r_{11} r_{12}}(r_{21}, r_1, r_2) + P_2 \cdot \int_0^{r_T} dr_{21} \int_0^{r_T} dr_{12}\, p_{r_{21} r_{12} r_{11} r_{22}}(r_{21}, r_{12}, r_1, r_2)$$

The joint PDF at the SSC combiner output at two time instants (Weibull)

$r_1 < r_T$, $r_2 < r_T$

(2)

$$p_{r_1 r_2}(r_1, r_2) = P_1 A(r_1, \beta_2, \Omega_2) A(r_2, \beta_1, \Omega_1) +$$

$$+ P_2 A(r_1, \beta_1, \Omega_1) A(r_2, \beta_2, \Omega_2)$$

where:

$$A(r, \beta, \Omega) = \frac{\beta}{\Omega} r^{\beta-1} e^{-\frac{r^{\beta}}{\Omega}} \left[1 - Q_1 \left(\frac{\sqrt{2\rho}}{\sqrt{\Omega(1-\rho)}} r^{\beta/2}, \frac{\sqrt{2}}{\sqrt{\Omega(1-\rho)}} r_t^{\beta/2} \right) \right]$$

The joint PDF at the SSC combiner output at two time instants

$r_1 \geq r_T$, $r_2 < r_T$

(2)

$$p_{r_1 r_2}(r_1, r_2) = P_1 A(r_1, \beta_1, \Omega_1) \frac{\beta_2}{\Omega_2} r_2^{\beta_2 - 1} e^{-\frac{r_2^{\beta_2}}{\Omega_2}} +$$

$$+ P_1 A(r_1, \beta_2, \Omega_2) A(r_2, \beta_1, \Omega_1) +$$

$$+ P_2 A(r_1, \beta_2, \Omega_2) \frac{\beta_1}{\Omega_1} r_2^{\beta_1 - 1} e^{-\frac{r_2^{\beta_1}}{\Omega_1}} + P_2 A(r_1, \beta_1, \Omega_1) A(r_2, \beta_2, \Omega_2)$$

The joint PDF at the SSC combiner output at two time instants

$r_1 < r_T,\ r_2 \geq r_T$

(2)

$$p_{r_1 r_2}(r_1, r_2) = P_1\left(1 - e^{-\frac{r_1^{\beta_1}}{\Omega_1}}\right)\frac{\beta_2^{\,2}(r_1 r_2)^{\beta_1 - 1}}{\Omega_2^{\,2}(1-\rho)} e^{-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_2}}{\Omega_2} + \frac{r_2^{\beta_2}}{\Omega_2}\right)} I_0\left[\frac{2\sqrt{\rho}\, r_1^{\beta_2/2} r_2^{\beta_2/2}}{(1-\rho)\Omega_2}\right] +$$

$$+ P_1 A(r_1, \beta_2, \Omega_2) A(r_2, \beta_1, \Omega_1) +$$

$$+ P_2\left(1 - e^{-\frac{r_1^{\beta_2}}{\Omega_2}}\right)\frac{\beta_1^{\,2}(r_1 r_2)^{\beta_1 - 1}}{\Omega_1^{\,2}(1-\rho)} e^{-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_1} + \frac{r_2^{\beta_1}}{\Omega_1}\right)} I_0\left[\frac{2\sqrt{\rho}\, r_1^{\beta_1/2} r_2^{\beta_1/2}}{(1-\rho)\Omega_1}\right] +$$

$$+ P_2 A(r_1, \beta_1, \Omega_1) A(r_2, \beta_2, \Omega_2)$$

The joint PDF at the SSC combiner output at two time instants

$r_1 \geq r_T,\ r_2 \geq r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \frac{\beta_1^{\,2}(r_1 r_2)^{\beta_1 - 1}}{\Omega_1^{\,2}(1-\rho)} e^{-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_1}+\frac{r_2^{\beta_1}}{\Omega_1}\right)} I_0\left[\frac{2\sqrt{\rho}\, r_1^{\beta_1/2} r_2^{\beta_1/2}}{(1-\rho)\Omega_1}\right] +$$

$$+ P_1 A(r_1, \beta_1, \Omega_1) \frac{\beta_2}{\Omega_2} r_2^{\beta_2 - 1} e^{-\frac{r_2^{\beta_2}}{\Omega_2}} + P_1 A(r_1, \beta_2, \Omega_2) A(r_2, \beta_1, \Omega_1) +$$

$$+ P_1\left(1 - e^{-\frac{r_t^{\beta_1}}{\Omega_1}}\right) \frac{\beta_2^{\,2}(r_1 r_2)^{\beta_1 - 1}}{\Omega_2^{\,2}(1-\rho)} e^{-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_2}}{\Omega_2}+\frac{r_2^{\beta_2}}{\Omega_2}\right)} I_0\left[\frac{2\sqrt{\rho}\, r_1^{\beta_2/2} r_2^{\beta_2/2}}{(1-\rho)\Omega_2}\right] +$$

$$+ P_2 \frac{\beta_1^{\,2}(r_1 r_2)^{\beta_1 - 1}}{\Omega_1^{\,2}(1-\rho)} e^{-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_1}+\frac{r_2^{\beta_1}}{\Omega_1}\right)} I_0\left[\frac{2\sqrt{\rho}\, r_1^{\beta_1/2} r_2^{\beta_1/2}}{(1-\rho)\Omega_1}\right] +$$

$$+ P_2 A(r_1, \beta_2, \Omega_2) \frac{\beta_1}{\Omega_1} r_2^{\beta_1 - 1} e^{-\frac{r_2^{\beta_1}}{\Omega_1}} + P_2 A(r_1, \beta_1, \Omega_1) A(r_2, \beta_2, \Omega_2) + \tag{2}$$

Joint PDF of two correlated signals

- The joint probability density function of correlated signals r_1 and r_2 with Gamma distribution and the same σ is

$$p_{r_1 r_2}(r_1, r_2) = \frac{\rho^{-\frac{c-1}{2}}}{\Gamma(c)(1-\rho)\Omega^{c+1}} (x_1 x_2)^{\frac{c-1}{2}} e^{-\frac{x_1 + x_2}{(1-\rho)\Omega}} I_{c-1}\left(\frac{2\sqrt{\rho x_1 x_2}}{(1-\rho)\Omega} \right)$$

Joint PDF of SSC combiner in the presence of Rayleigh fading

Za $r_1 < r_T,\ r_2 < r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \frac{r_1}{\sigma_2^2} e^{-\frac{r_1^2}{2\sigma_2^2}} \frac{r_2}{\sigma_1^2} e^{-\frac{r_2^2}{2\sigma_1^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_2\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2\sqrt{1-\rho^2}}\right)\right]\left[1 - Q_1\left(\frac{\rho r_2}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right] +$$

$$+ P_2 \frac{r_1}{\sigma_1^2} e^{-\frac{r_1^2}{2\sigma_1^2}} \frac{r_2}{\sigma_2^2} e^{-\frac{r_2^2}{2\sigma_2^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right]\left[1 - Q_1\left(\frac{\rho r_2}{\sigma_2\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2\sqrt{1-\rho^2}}\right)\right]$$

Za $r_1 \geq r_T,\ r_2 < r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \frac{r_1}{\sigma_1^2} e^{-\frac{r_1^2}{2\sigma_1^2}} \frac{r_2}{\sigma_2^2} e^{-\frac{r_2^2}{2\sigma_2^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right] +$$

$$+ P_1 \frac{r_1}{\sigma_2^2} e^{-\frac{r_1^2}{2\sigma_2^2}} \frac{r_2}{\sigma_1^2} e^{-\frac{r_2^2}{2\sigma_1^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_2\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2\sqrt{1-\rho^2}}\right)\right]\left[1 - Q_1\left(\frac{\rho r_2}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right] +$$

$$+ P_2 \frac{r_1}{\sigma_2^2} e^{-\frac{r_1^2}{2\sigma_2^2}} \frac{r_2}{\sigma_1^2} e^{-\frac{r_2^2}{2\sigma_1^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_2\sqrt{1-\rho^2}}, r_T \sigma_2\sqrt{1-\rho^2}\right)\right] +$$

$$+ P_2 \frac{r_1}{\sigma_1^2} e^{-\frac{r_1^2}{2\sigma_1^2}} \frac{r_2}{\sigma_2^2} e^{-\frac{r_2^2}{2\sigma_2^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right]\left[1 - Q_1\left(\frac{\rho r_2}{\sigma_2\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2\sqrt{1-\rho^2}}\right)\right]$$

Joint PDF of SSC combiner in the presence of Rayleigh fading

Za $r_1 < r_T$, $r_2 \geq r_T$

$$
\begin{aligned}
p_{r_1 r_2}(r_1, r_2) = {} & P_1 \left(1 - e^{-\frac{r_T}{2\sigma_1^2}} \right) \frac{r_1 r_2}{\sigma_2^4 (1-\rho^2)} e^{-\frac{r_1^2 + r_2^2}{2\sigma_2^2 (1-\rho^2)}} I_0 \left(\frac{\rho r_1 r_2}{\sigma_2^2 (1-\rho^2)} \right) + \\
& + P_1 \frac{r_1}{\sigma_2^2} e^{-\frac{r_1^2}{2\sigma_2^2}} \frac{r_2}{\sigma_1^2} e^{-\frac{r_2^2}{2\sigma_1^2}} \left[1 - Q_1 \left(\frac{\rho r_1}{\sigma_2 \sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2 \sqrt{1-\rho^2}} \right) \right] \left[1 - Q_1 \left(\frac{\rho r_2}{\sigma_1 \sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1 \sqrt{1-\rho^2}} \right) \right] + \\
& + P_2 \left(1 - e^{-\frac{r_T}{2\sigma_2^2}} \right) \frac{r_1 r_2}{\sigma_1^4 (1-\rho^2)} e^{-\frac{r_1^2 + r_2^2}{2\sigma_1^2 (1-\rho^2)}} I_0 \left(\frac{\rho r_1 r_2}{\sigma_1^2 (1-\rho^2)} \right) + \\
& + P_2 \frac{r_1}{\sigma_1^2} e^{-\frac{r_1^2}{2\sigma_1^2}} \frac{r_2}{\sigma_2^2} e^{-\frac{r_2^2}{2\sigma_2^2}} \left[1 - Q_1 \left(\frac{\rho r_1}{\sigma_1 \sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1 \sqrt{1-\rho^2}} \right) \right] \left[1 - Q_1 \left(\frac{\rho r_2}{\sigma_2 \sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2 \sqrt{1-\rho^2}} \right) \right]
\end{aligned}
$$

Joint PDF of SSC combiner in the presence of Rayleigh fading

Za $r_1 \geq r_T$, $r_2 \geq r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 \frac{r_1 r_2}{\sigma_1^{\ 4}(1-\rho^2)} e^{-\frac{r_1^2 + r_2^2}{2\sigma_1^2 (1-\rho^2)}} I_0\left(\frac{\rho r_1 r_2}{\sigma_1^{\ 2}(1-\rho^2)}\right) +$$

$$+ P_1 \frac{r_1}{\sigma_1^{\ 2}} e^{-\frac{r_1^2}{2\sigma_1^2}} \frac{r_2}{\sigma_2^{\ 2}} e^{-\frac{r_2^2}{2\sigma_2^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right] +$$

$$+ P_1 \left(1 - e^{-\frac{r_T}{2\sigma_1^2}}\right) \frac{r_1 r_2}{\sigma_2^{\ 4}(1-\rho^2)} e^{-\frac{r_1^2 + r_2^2}{2\sigma_2^2 (1-\rho^2)}} I_0\left(\frac{\rho r_1 r_2}{\sigma_2^{\ 2}(1-\rho^2)}\right) +$$

$$+ P_1 \frac{r_1}{\sigma_2^{\ 2}} e^{-\frac{r_1^2}{2\sigma_2^2}} \frac{r_2}{\sigma_1^{\ 2}} e^{-\frac{r_2^2}{2\sigma_1^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_2\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2\sqrt{1-\rho^2}}\right)\right]\left[1 - Q_1\left(\frac{\rho r_2}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right] +$$

$$+ P_2 \frac{r_1 r_2}{\sigma_2^{\ 4}(1-\rho^2)} e^{-\frac{r_1^2 + r_2^2}{2\sigma_2^2 (1-\rho^2)}} I_0\left(\frac{\rho r_1 r_2}{\sigma_2^{\ 2}(1-\rho^2)}\right) +$$

$$+ P_2 \frac{r_1}{\sigma_2^{\ 2}} e^{-\frac{r_1^2}{2\sigma_2^2}} \frac{r_2}{\sigma_1^{\ 2}} e^{-\frac{r_2^2}{2\sigma_1^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_2\sqrt{1-\rho^2}}, r_T \sigma_2 \sqrt{1-\rho^2}\right)\right] +$$

$$+ P_2 \left(1 - e^{-\frac{r_T}{2\sigma_2^2}}\right) \frac{r_1 r_2}{\sigma_1^{\ 4}(1-\rho^2)} e^{-\frac{r_1^2 + r_2^2}{2\sigma_1^2 (1-\rho^2)}} I_0\left(\frac{\rho r_1 r_2}{\sigma_1^{\ 2}(1-\rho^2)}\right) +$$

$$+ P_2 \frac{r_1}{\sigma_1^{\ 2}} e^{-\frac{r_1^2}{2\sigma_1^2}} \frac{r_2}{\sigma_2^{\ 2}} e^{-\frac{r_2^2}{2\sigma_2^2}} \left[1 - Q_1\left(\frac{\rho r_1}{\sigma_1\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_1\sqrt{1-\rho^2}}\right)\right]\left[1 - Q_1\left(\frac{\rho r_2}{\sigma_2\sqrt{1-\rho^2}}, \frac{r_T}{\sigma_2\sqrt{1-\rho^2}}\right)\right]$$

PDF of Derivatives at the output of the SSC combiner in the presence of Rayleigh fading at two time instants

(2)
$$p_{\dot r_1}(\dot r_1) = P_1\frac{1}{\sqrt{2\pi}\dot\sigma_1}e^{-\frac{\dot r_1^2}{2\dot\sigma_1^2}} + P_2\frac{1}{\sqrt{2\pi}\dot\sigma_2}e^{-\frac{\dot r_1^2}{2\dot\sigma_2^2}} + \left(P_2\left(1-e^{-\frac{r_T^2}{2\sigma_2^2}}\right) - P_1\left(1-e^{-\frac{r_T^2}{2\sigma_1^2}}\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_1}e^{-\frac{\dot r_1^2}{2\dot\sigma_1^2}} +$$
$$+\left(P_1\left(1-e^{-\frac{r_T^2}{2\sigma_1^2}}\right) - P_2\left(1-e^{-\frac{r_T^2}{2\sigma_2^2}}\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_2}e^{-\frac{\dot r_1^2}{2\dot\sigma_2^2}}$$

$$p_{\dot r_2}(\dot r_2) = P_1\left(1-e^{-\frac{r_T^2}{2\sigma_1^2}}\right)\left(1-e^{-\frac{r_T^2}{2\sigma_2^2}}\right)\frac{1}{\sqrt{2\pi}\dot\sigma_1}e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + P_2\left(1-e^{-\frac{r_T^2}{2\sigma_1^2}}\right)\left(1-e^{-\frac{r_T^2}{2\sigma_2^2}}\right)\frac{1}{\sqrt{2\pi}\dot\sigma_2}e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} +$$
$$+P_1B_1(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_2}e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} + P_2B_2(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_1}e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + P_1\left(1-e^{-\frac{r_T^2}{2\sigma_1^2}}\right)\left(1-\left(1-e^{-\frac{r_T^2}{2\sigma_2^2}}\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_2}e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}} +$$
$$+P_2\left(1-e^{-\frac{r_T^2}{2\sigma_2^2}}\right)\left(1-\left(1-e^{-\frac{r_T^2}{2\sigma_1^2}}\right)\right)\frac{1}{\sqrt{2\pi}\dot\sigma_1}e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + P_1C_1(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_1}e^{-\frac{\dot r_2^2}{2\dot\sigma_1^2}} + P_2C_2(r_T)\frac{1}{\sqrt{2\pi}\dot\sigma_2}e^{-\frac{\dot r_2^2}{2\dot\sigma_2^2}}$$

$$B_i(r_T) = -e^{-\frac{r_T^2}{2\sigma_i^2}}\left[1-Q_1\left(\frac{r_T}{\sigma_i}\sqrt{\frac{1}{(1-\rho_i^2)}}, \frac{\rho_i r_T}{\sigma_i\sqrt{1-\rho_i^2}}\right)\right] + e^{-\frac{r_T^2}{2\sigma_i^2}}\left[1-Q_1\left(\frac{\rho_i r_T}{\sigma_i\sqrt{1-\rho_i^2}}, \frac{r_T}{\sigma_i\sqrt{1-\rho_i^2}}\right)\right]$$

$$C_i(r_T) = e^{-\frac{r_T^2}{2\sigma_i^2}}\left[1-Q_1\left(\frac{r_T}{\sigma_i}\sqrt{\frac{1}{(1-\rho_i^2)}}, \frac{\rho_i r_T}{\sigma_i\sqrt{1-\rho_i^2}}\right)\right] + e^{-\frac{r_T^2}{2\sigma_i^2}}Q_1\left(\frac{\rho_i r_T}{\sigma_i\sqrt{1-\rho_i^2}}, \frac{r_T}{\sigma_i\sqrt{1-\rho_i^2}}\right)$$

Rice fading is present:

for $r_1 < r_T$, $r_2 < r_T$ it is:

$$p_{r_1 r_2}(r_1, r_2) = P_1 C_1(r_1, \sigma_2, A_2) C_1(r_2, \sigma_1, A_1) +$$

$$+ P_2 C_1(r_1, \sigma_1, A_1) C_1(r_2, \sigma_2, A_2)$$

For $r_1 \geq r_T$, $r_2 < r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1 C_1(r_1, \sigma_1, A_1) \frac{r_2}{\sigma_2^{\ 2}} e^{-\frac{r_2^{\ 2} + A_2^{\ 2}}{2\sigma_2^{\ 2}}} I_0\left(\frac{r_2 A_2}{\sigma_2^{\ 2}}\right) +$$

$$+ P_1 C_1(r_1, \sigma_2, A_2) C_1(r_2, \sigma_1, A_1) +$$

$$+ P_2 C_1(r_1, \sigma_2, A_2) \frac{r_2}{\sigma_1^{\ 2}} e^{-\frac{r_2^{\ 2} + A_1^{\ 2}}{2\sigma_1^{\ 2}}} I_0\left(\frac{r_2 A_1}{\sigma_1^{\ 2}}\right) +$$

$$+ P_2 C_1(r_1, \sigma_1, A_1) C_1(r_2, \sigma_2, A_2)$$

For $r_1 < r_T$, $r_2 \geq r_T$

$$p_{r_1 r_2}(r_1, r_2) = P_1\left(1 - Q_1\left(A/\sigma_1, r_t/\sigma_1\right)\right) C_2(r_1, r_2, \sigma_2, A_2) +$$

$$+ P_2\left(1 - Q_1\left(A/\sigma_2, r_t/\sigma_2\right)\right) C_2(r_1, r_2, \sigma_1, A_1) +$$

$$+ P_1 C_1(r_1, \sigma_2, A_2) C_1(r_2, \sigma_1, A_1) +$$

$$+ P_2 C_1(r_1, \sigma_1, A_1) C_1(r_2, \sigma_2, A_2)$$

For $r_1 \geq r_T$, $r_2 \geq r_T$

$$
\begin{aligned}
p_{r_1 r_2}(r_1, r_2) = {} & P_1 C_2(r_1, r_2, \sigma_1, A_1) + \\
& + P_1 C_1(r_1, \sigma_1, A_1) \frac{r_2}{\sigma_2^2} e^{-\frac{r_2^2 + A_2^2}{2\sigma_2^2}} I_0\left(\frac{r_2 A_2}{\sigma_2^2}\right) + \\
& + P_1 C_1(r_1, \sigma_2, A_2) C_1(r_2, \sigma_1, A_1) + \\
& + P_1\left(1 - Q_1\left(A/\sigma_1, r_t/\sigma_1\right)\right) C_2(r_1, r_2, \sigma_2, A_2) + \\
& + P_2 C_1(r_1, \sigma_2, A_2) \frac{r_2}{\sigma_1^2} e^{-\frac{r_2^2 + A_1^2}{2\sigma_1^2}} I_0\left(\frac{r_2 A_1}{\sigma_1^2}\right) + \\
& + P_2 C_2(r_1, r_2, \sigma_2, A_2) + \\
& + P_2 C_1(r_1, \sigma_1, A_1) C_1(r_2, \sigma_2, A_2) + \\
& + P_2\left(1 - Q_1\left(A/\sigma_2, r_t/\sigma_2\right)\right) C_2(r_1, r_2, \sigma_1, A_1)
\end{aligned}
$$

- The expressions for probability density functions and joint probability density functions for SSC combiner output signals at two time instants in the presence of different fading distributions are determined and then, these expressions are used for calculation of system performances, such as the bit error rate and the outage probability

- The level crossing rate and the average fade duration are also very often used in designing of wireless communication systems as measures for their quality.
- To obtain second order system characteristics the expressions for signal derivatives are needed
- Because of this, the probability density functions of derivatives in two time instants for SSC combiner in fading channels have to be determined

Outage probability for the complex combiner

- Outage probability is defined as the probability that the combiner output signal value falls below a given threshold r_{th}

$$P_{out}(r_{th}) = \int_0^{r_{th}} [p_1(r) + p_2(r) + p_3(r) + p_4(r)]dr$$

Amount of fading for the SSC/MRC combiner

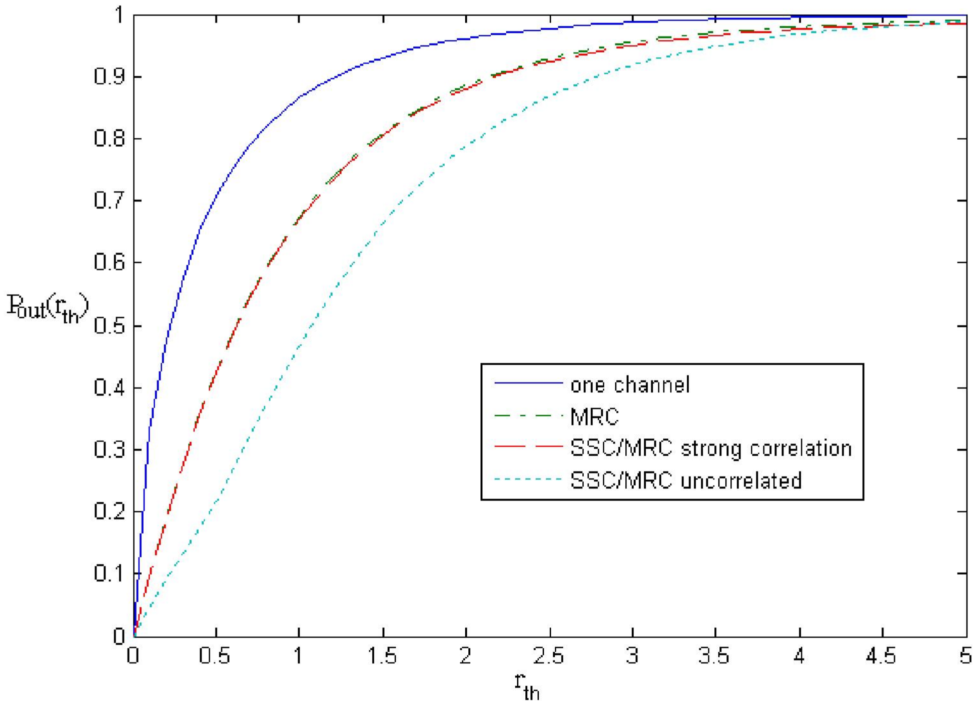
- Amount of fading (AF) is a unified measure of the severity of fading for particular channel model

$$AF = \frac{\int_0^\infty [p_1(r) + p_2(r) + p_3(r) + p_4(r)] r^2 dr}{\left(\int_0^\infty [p_1(r) + p_2(r) + p_3(r) + p_4(r)] r dr\right)^2} - 1$$

The bit error rate (BER)

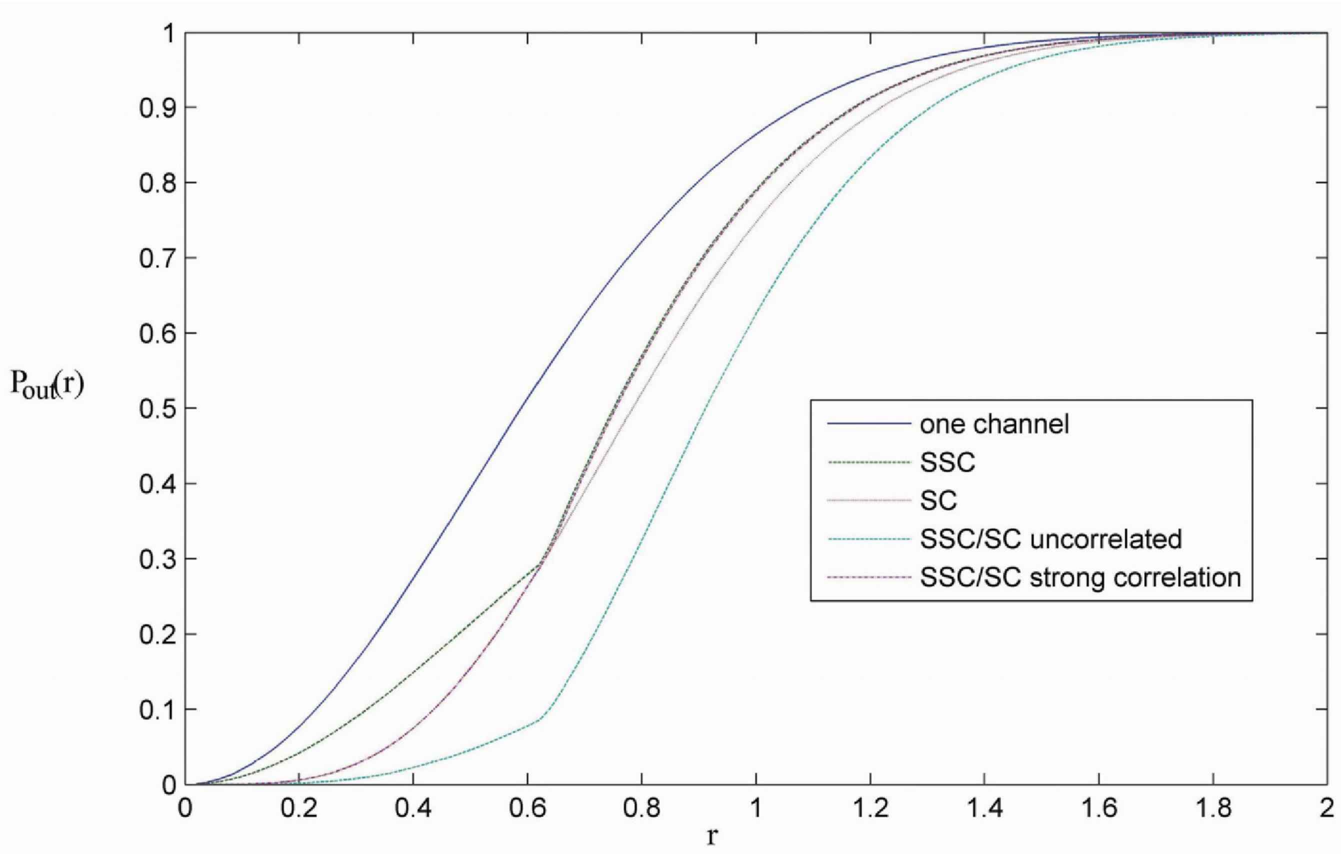
$$P_b(e) = \underbrace{\int_0^\infty \int_0^\infty \cdots \int_0^\infty}_{L} P_b\left(\{r_l\}_{l=1}^{L}\right) \prod_{l=1}^{L} p_{r_1,r_2,\ldots r_L}(r_1, r_2, \ldots r_L)\, dr_1\, dr_2 \cdots dr_L$$

Outage probability for the SSC/MRC combiner – Weibull

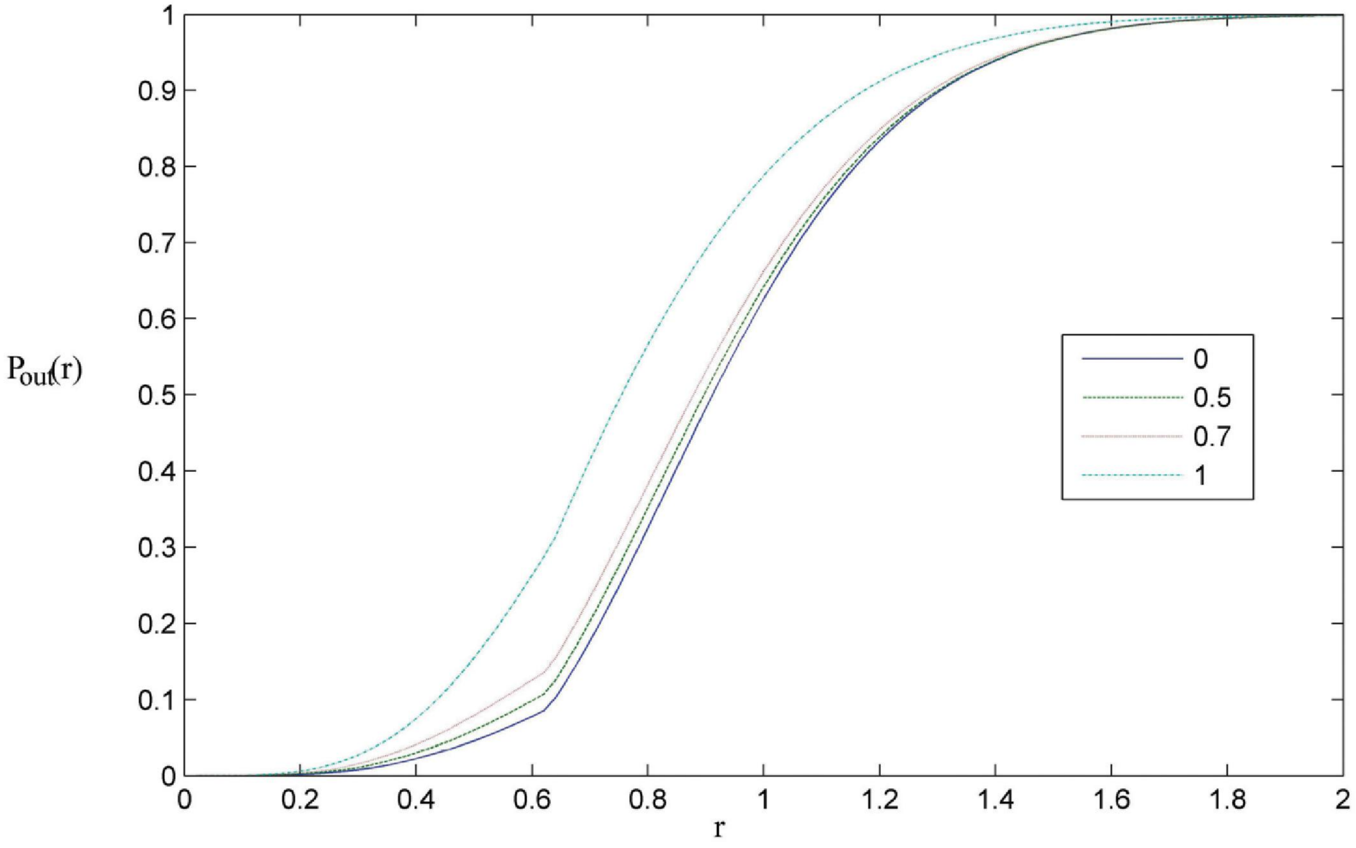


Outage probability for different types of combiners for parameters β=0.7 and Ω=0.5

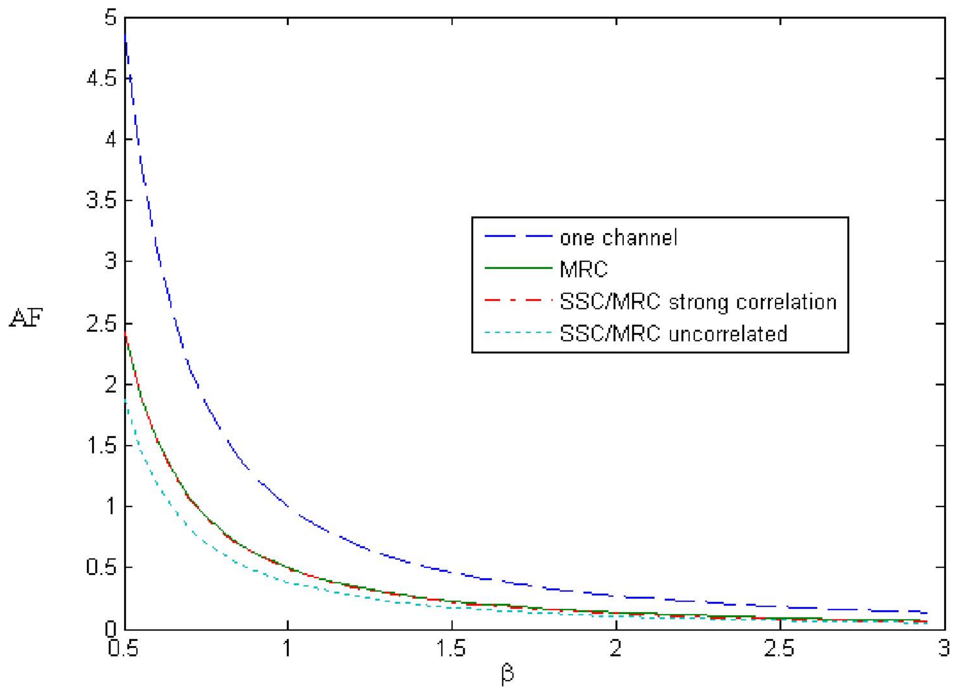
Outage probability for the different combiners – Rayleigh



Outage probability for the different combiners – Rayleigh

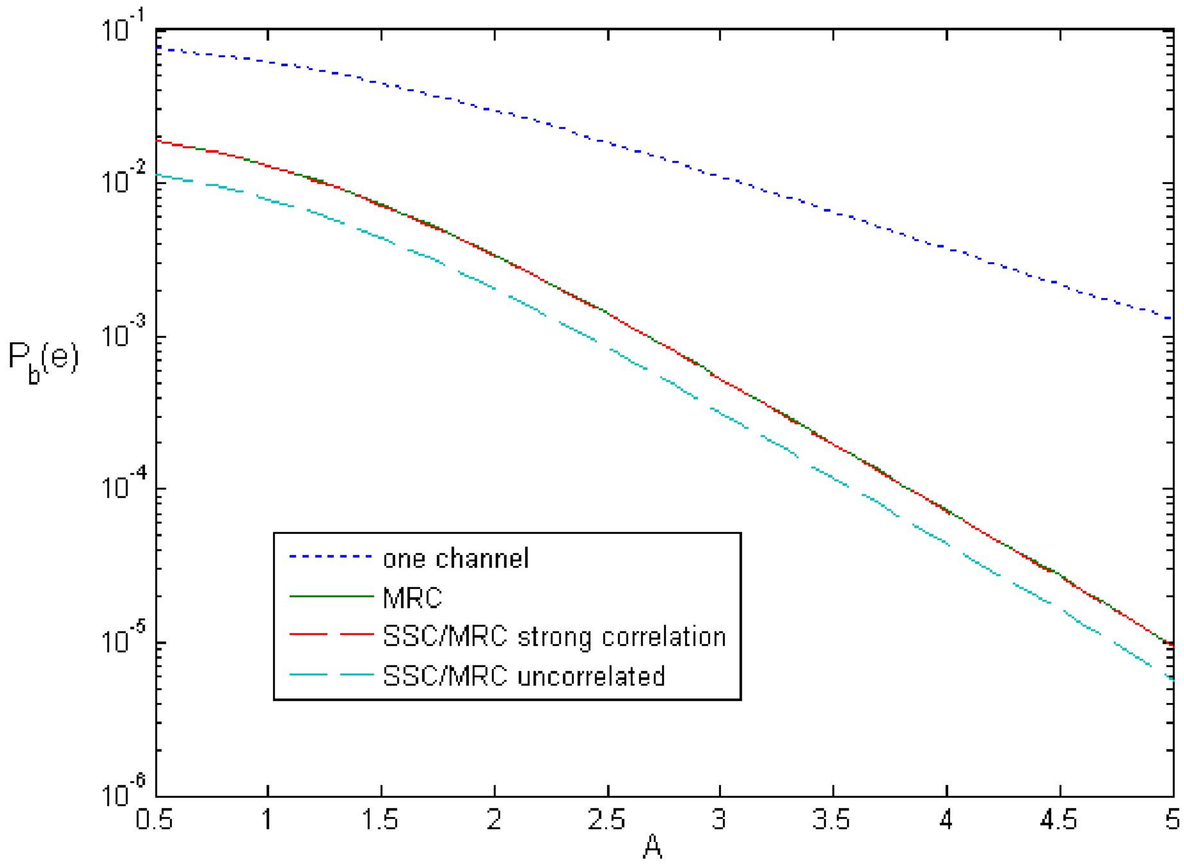


Amount of fading for the SSC/MRC combiner- Weibull



Amount of fading for different types of combiners for Ω=0.5

Bit error rate for different types of combiners – Rice fading



Conclusion

- The combining techniques like SSC and MRC are simple and frequently used for signals combining in diversity systems for reducing fading effects
- The joint probability density function of the complex dual SSC/MRC combiner output signal at two time instants, in the presence of different fading distributions are determined

Conclusion

- It is presented the improvement of characteristics of complex SSC/SC(MRC) combiner at two time instants comparing with classical SSC and MRC combiners

Conclusion

- Complex SSC/MRC combiner is not economical in the case of strongly correlated signals because it does not give better performance than MRC combiner

Acknowledgment

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