Stability and control of power grids with diluted network topology

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Presenter's short resume

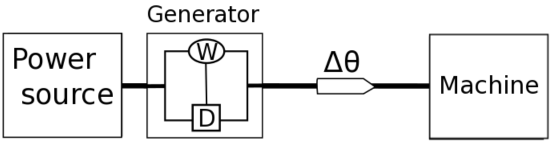
- **B.Sc. in physics** from Technical University of Berlin
  - Period: april 2013 - february 2016
  - Focus: nonlinear dynamics ⇒ Stuart-Landau oscillators
  - Thesis: Stability of amplitude chimeras in the Stuart-Landau model
- **M.Sc. in physics** from Technical University of Berlin
  - Period: february 2016 - march 2018
  - Focus: nonlinear dynamics ⇒ power grid networks
  - Thesis: Influence of noise in shaping the dynamics of power grids
- **PhD in applied mathematics** from University Grenoble Alpes
  - Period: september 2018 - august 2021
  - Focus: control theory ⇒ control of urban traffic ⇒ PDEs
  - Thesis: Traffic control in large-scale urban networks

Motivation

- Ecological **problem**: traditional power plants
  $\Rightarrow$ emission of greenhouse gases $\Rightarrow$ global warming
- Suggested **solution**: increased usage of renewable energy sources
- Three major **changes**: decentralization, spatial separation and fluctuations of the power output
- Main **question**: effect on stable operation of power grids?

Modeling power grids

Illustration of the power grid mechanism



- **Power plant** consists of power source and generator
  $\Rightarrow$ electrical power is produced with frequency close to $\Omega = 50$ Hz
- $\Delta\theta$: **phase difference** between the active generator and the passive machine

Kuramoto model = power balance equation for generator/machine

$$P_{source} = P_{acc} + P_{diss} + P_{transmitted}$$

Model

Extended Kuramoto model

Dynamic model for rotator i

$$m\ddot{\theta}_i + \dot{\theta}_i = \Omega_i + \frac{K}{N_i}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i)$$

- θ_i, $\dot{\theta}_i$: instantaneous phase and frequency
- Ω_i: natural frequency (power of generators and loads)
- K: coupling constant (transmission line capacities)
- $m > 0$: inertial mass
- A: symmetric connectivity matrix
- $N_i = N_c$: node degree (here constant)

  $N_c = p(N-1)$, where p is dilution parameter, $0 < p < 1$

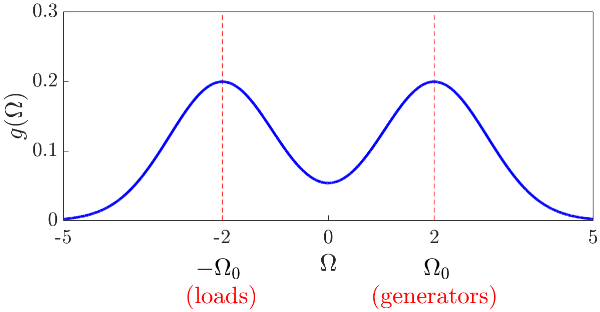
Bimodal frequency distribution

Gaussian bimodal distribution

$$\Omega_i : \qquad g(\Omega) = \left[\frac{p_g}{\sqrt{2\pi}} e^{-\frac{(\Omega-\Omega_{0^+})^2}{2}} + \frac{1-p_g}{\sqrt{2\pi}} e^{-\frac{(\Omega+\Omega_{0^-})^2}{2}} \right]$$

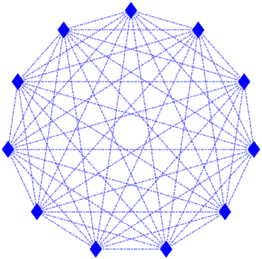
and p_g determines the peak location

- Symmetric (case below): $p_g = 0.5$, $\Omega_{0^+} = \Omega_{0^-} = 2$
- Asymmetric: $p_g = 0.2$, $\Omega_{0^+} = 3.2$, $\Omega_{0^-} = 0.8$



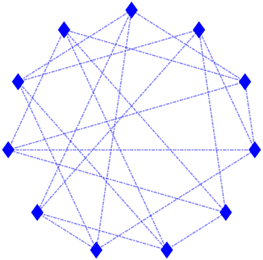
Globally coupled and diluted networks

Fully connected network:
maximal node degree $N_c = N - 1$



p=1.0, no disorder

Erdős–Rényi network:
$N_c = p(N - 1)$ with $p < 1$



p=0.40, **more realistic**

Our contributions and presentation plan

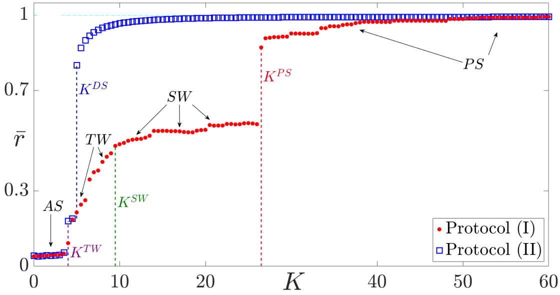
- **Stable operation** of power grids = **synchronous** state of the network
- **Stability analysis** of sparse power grid networks
- **Numerical solution** to find frequency synchronized solutions
- **Control** of unstable frequency synchronized solutions
- Examine interplay of **noise** and random connectivity
- Control of noisy **Italian power grid**

Deterministic system

Synchronization transition: order parameter

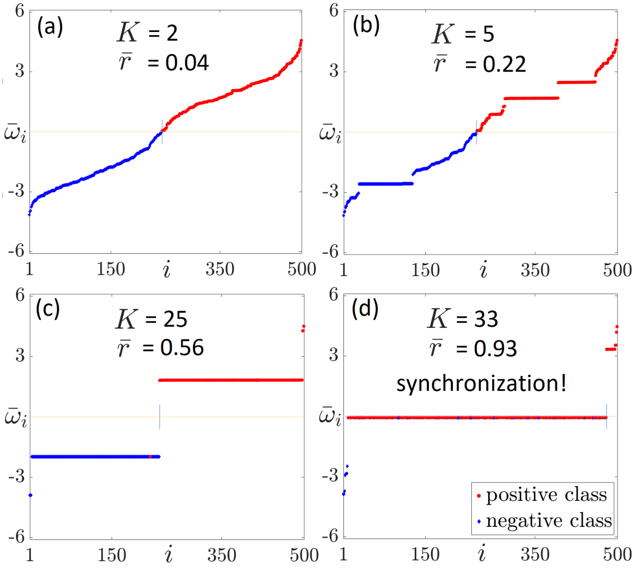
Complex order parameter: $r(t)e^{i\phi(t)} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}, \qquad r(t) \in [0, 1]$

Parameters: $m = 6$, $p = 0.2$, $\Omega_0 = 2$, $N = 500$



AS: Asynchronous State
TW: Traveling Wave
SW: Standing Wave
PS: Partial Synchronization

Synchronization transition: frequency profile



Kuramoto model in case of frequency synchronization

- **Frequency synchronization**: $\dot{\theta}_1 = \ldots = \dot{\theta}_n = \omega_c \Rightarrow \theta_i - \theta_j = const$
- Common frequency:

$$\omega_c = \frac{1}{N} \sum_{i=1}^{N} \Omega_i \quad \Rightarrow \quad \text{for large networks we expect } \ \omega_c \approx 0$$

Frequency synchronized solution of Kuramoto model

$$\sigma \sum_{j=1}^{N} A_{ij} \sin(\theta_j^0 - \theta_i^0) = -\alpha(\Omega_i - \omega_c)$$

where $\alpha = \frac{1}{m}$ and $\sigma = \frac{K}{N_c m}$

Stability analysis of frequency synchronized solution

Linearized system for small perturbations from frequency synchronization:

$$\begin{pmatrix} \delta\dot{\theta} \\ \delta\dot{\omega} \end{pmatrix} = \begin{bmatrix} \mathbf{0} & \mathbb{I} \\ \sigma\mathbf{M} & -\alpha\mathbb{I} \end{bmatrix} \begin{pmatrix} \delta\theta \\ \delta\omega \end{pmatrix}, \quad \text{where } \delta\theta, \delta\omega : N\text{-dim vectors}$$

M: $N \times N$ Laplacian matrix of a weighted undirected graph

$$\mathbf{M} = \begin{bmatrix} -\sum_j A_{1j}\cos(\theta_j^0 - \theta_1^0) & \cdots & A_{1N}\cos(\theta_1^0 - \theta_N^0) \\ \cdots & \cdots & \cdots \\ A_{N1}\cos(\theta_N^0 - \theta_1^0) & \cdots & -\sum_j A_{Nj}\cos(\theta_j^0 - \theta_N^0) \end{bmatrix}$$

Solve the eigenvalue problem and obtain:

$$\left|(\lambda^2 + \lambda\alpha)\mathbb{I} - \sigma\mathbf{M}\right| = 0 \quad \Rightarrow \quad \lambda = \frac{-\alpha \pm \sqrt{\alpha^2 + 4\mu\sigma}}{2}, \quad \text{with } \mu = eig(\mathbf{M})$$

If **M** is stable, then frequency synchronized solution is **stable**!

Finding solutions

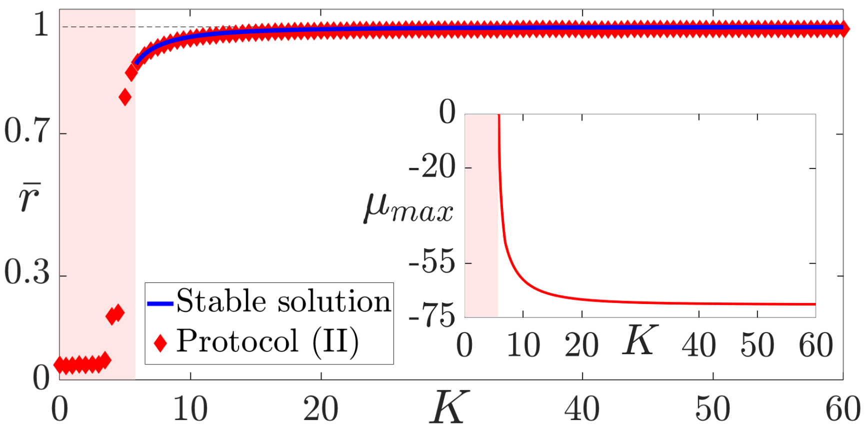
- If $\left|\theta_j^0 - \theta_i^0\right| < \frac{\pi}{2}$, then **M** is a *diagonally dominant matrix* $\Rightarrow \mu \leq 0$
- In the neighbourhood $\left\{\theta_i^0\right\} = 0$, solution is always stable ($\mu \leq 0$)
- Not for all coupling K there is a solution (only if $K > K_c$)
- Find stable and unstable solutions (set of initial phases) numerically:

$$\mathrm{F}_i(\theta^*) = \alpha(\Omega_i - \omega_c) + \sigma \sum_{j=1}^{N} A_{ij} \sin(\theta_j^* - \theta_i^*) = 0$$

- Use **Levenberg-Marquardt algorithm** (combination of Gauss-Newton and gradient descent) to minimize $\sum |F_i|^2$
- To find *critical coupling* K_c, we set $\theta_i^* \equiv 0$ as initial guess

Critical coupling K_c

Parameter set: $m = 6$, $p = 0.2$, $N = 500$

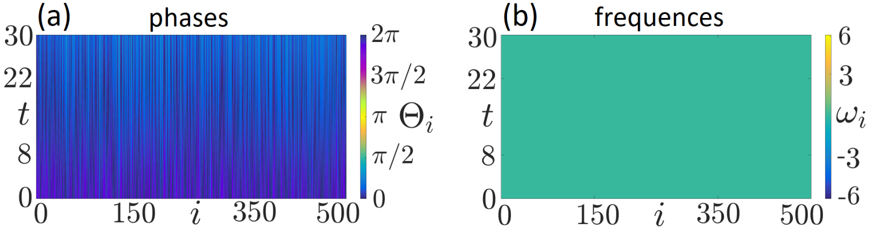


No solution can be found for $K < K_c = 5.8$

Stable solution

Stable frequency synchronized solution found for:

$K = 10$, $Re(\lambda_{max}) = -0.083$

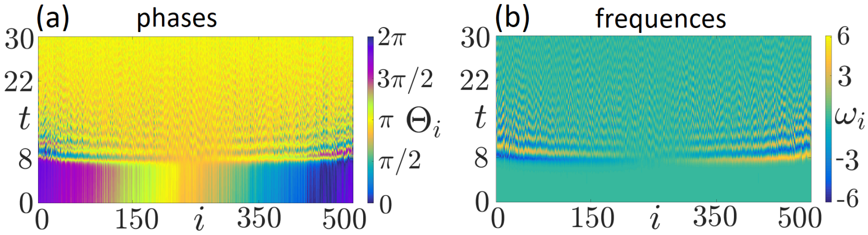


Narrow spectrum of initial phases

Unstable solution

Unstable frequency synchronized solution found for:

$K = 70$, $Re(\lambda_{max}) = 2.41$



Wide spectrum of initial phases

Behaviour is lost after $t = 9$ seconds $\Rightarrow$ we can stabilize it by control

Control: linear quadratic regulator (LQR)

- Add external control u_i to the Kuramoto system

$$\begin{cases} \dot{\theta}_i = \omega_i \\ \dot{\omega}_i = \alpha\Omega_i - \alpha\omega_i + \sigma \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i) + u_i \end{cases}$$

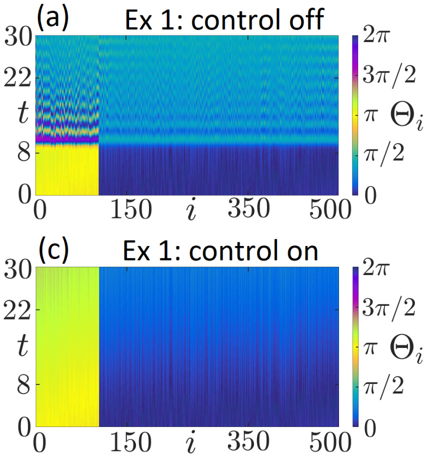
- Rewrite the *linearised* matrix system as

$$\begin{pmatrix} \delta\dot{\theta} \\ \delta\dot{\omega} \end{pmatrix} = \begin{bmatrix} \mathbf{0} & \mathbb{I} \\ \sigma\mathbf{M} & -\alpha\mathbb{I} \end{bmatrix} \begin{pmatrix} \delta\theta \\ \delta\omega \end{pmatrix} + \mathbf{B}u, \quad \text{where} \quad \mathbf{B} = \begin{bmatrix} \mathbb{O}_N \\ \mathbb{I}_N \end{bmatrix} \in R^{2N\times N}$$

- Let $u = -\mathbf{K}\begin{pmatrix} \delta\theta \\ \delta\omega \end{pmatrix}$. Use LQR to find **K** minimizing *quadratic* cost

$$J(u) = \int_0^\infty \left\| \begin{pmatrix} \delta\theta(t) \\ \delta\omega(t) \end{pmatrix} \right\|^2 + \left\| u(t) \right\|^2 dt \quad \Rightarrow \quad \text{non-positive eigenvalues}$$

Control of unstable states: stabilization



With control we obtain stable solutions for a large range of initial phases!

Stochastic system

Influence of temporal power fluctuations

Strong temporal **fluctuations** are a signature of renewable energy-based power grids, e.g., wind turbines

Extended Kuramoto model with noise

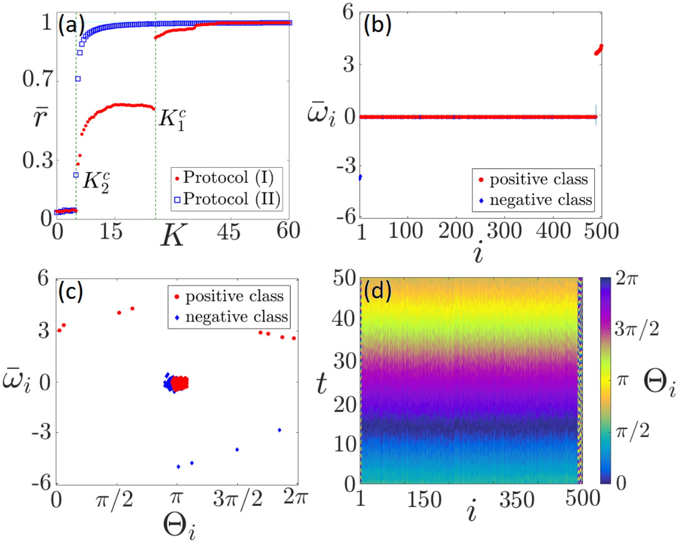
$$m\ddot{\theta}_i + \dot{\theta}_i = \Omega_i + \frac{K}{N_c}\sum_{j=1}^{N} A_{ij}\sin(\theta_j - \theta_i) + \sqrt{2D}\xi_i(t)$$

- ξ_i: Gaussian white noise, $\langle \xi_i \rangle = 0$ and $\langle \xi_i(t)\xi_j(s) \rangle = \delta_{ij}\delta(t-s)$
- D: noise intensity

Our goal: study the influence of noise on synchronization transition

Synchronization of power grids: small noise intensity

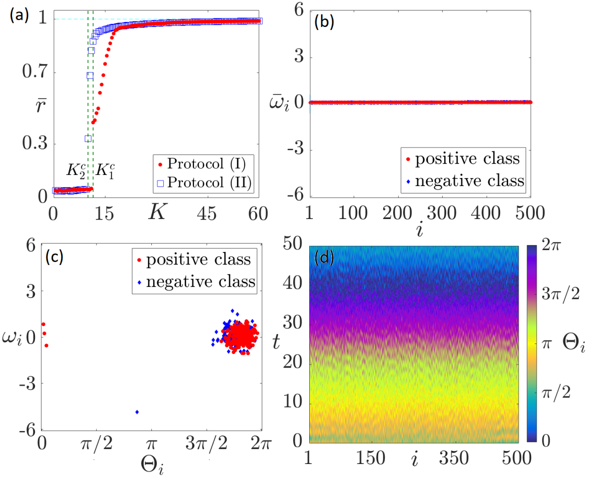
Parameters: $m = 6$, $p = 0.1$, $N = 500$, $\Omega_0 = 2$, $\sqrt{\mathbf{2D}} = \mathbf{5}$



Synchronization of power grids: large noise intensity

Parameters: $m = 6$, $p = 0.1$, $N = 500$, $\Omega_0 = 2$, $\boldsymbol{\sqrt{2D} = 15}$

Hysteretic region decreases with noise, **synchronization** occurs **earlier**

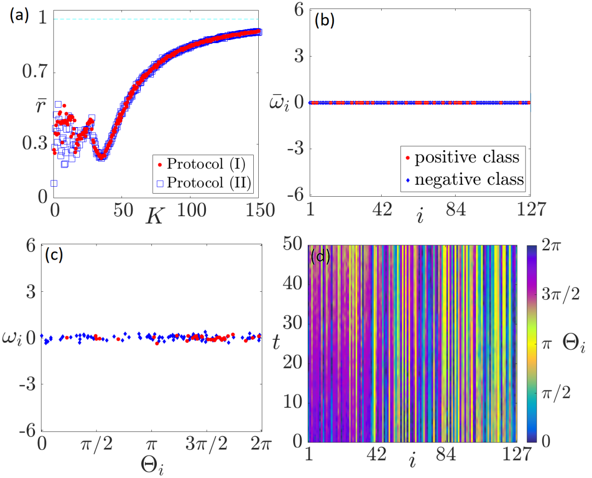


Synchronization of Italian power grid: low noise

Italian power grid: $N = 127$ nodes = 34 sources + 93 consumers (342 links)

Noise intensity: $\sqrt{\mathbf{2D}} = \mathbf{5}$

(b),(c),(d) $K = 30$

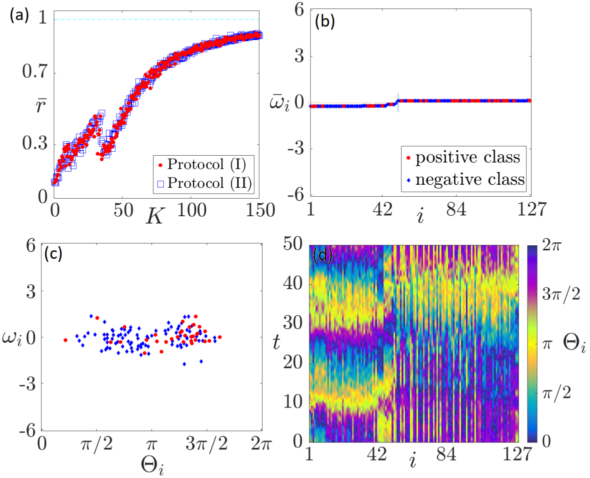


Synchronization of Italian power grid: high noise

Noise intensity: $\sqrt{\mathbf{2D}} = \mathbf{15}$

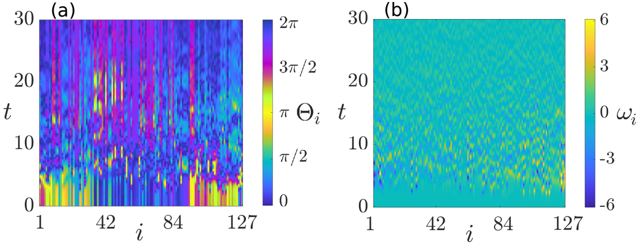
(b),(c),(d) $K = 30$

No constructive role of noise any more!

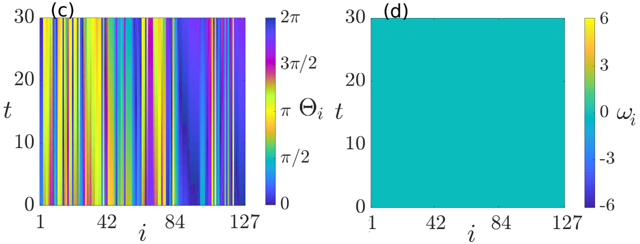


Control of noisy Italian power grid

Parameters: $K = 70$, $\sqrt{\mathbf{2D}} = \mathbf{5}$



(a),(b): control off, $\lambda_{max} = 4.01$ (c),(d): control on, $\lambda_{ctrl} = -0.867$



Conclusions

- **Stability analysis** enabled to characterize stable and unstable frequency synchronization solutions (numerical method: Levenberg-Marquardt)
- **No solutions** exist for small K, and unstable solutions are found for **large spectrum** of initial phases
- Linear quadratic regulator was applied to **stabilize** unstable solutions
- With **noise**, hysteretic region shrinks and intermediate states vanish
- Noise plays a **constructive role** on synchronization if spatial disorder is not too high
- **Italian grid** topology was examined under the influence of noise, and stabilized for low noise intensities

Thank you for attention!

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