A Knowledge Extraction from Epidemic Control Simulation

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His research interests include Finance and Agent-based Modeling.

1 min. Summary

- Purpose:
  Extract knowledge from epidemic control simulation
- Background:
  Large epidemics also affecting society and economy, other than public health.
- Approach:
  Apply the data envelopment analysis (DEA) for simulation outcome classification.
  Find which control policies are good or bad, under which kind of efficiency perspective?

Purpose

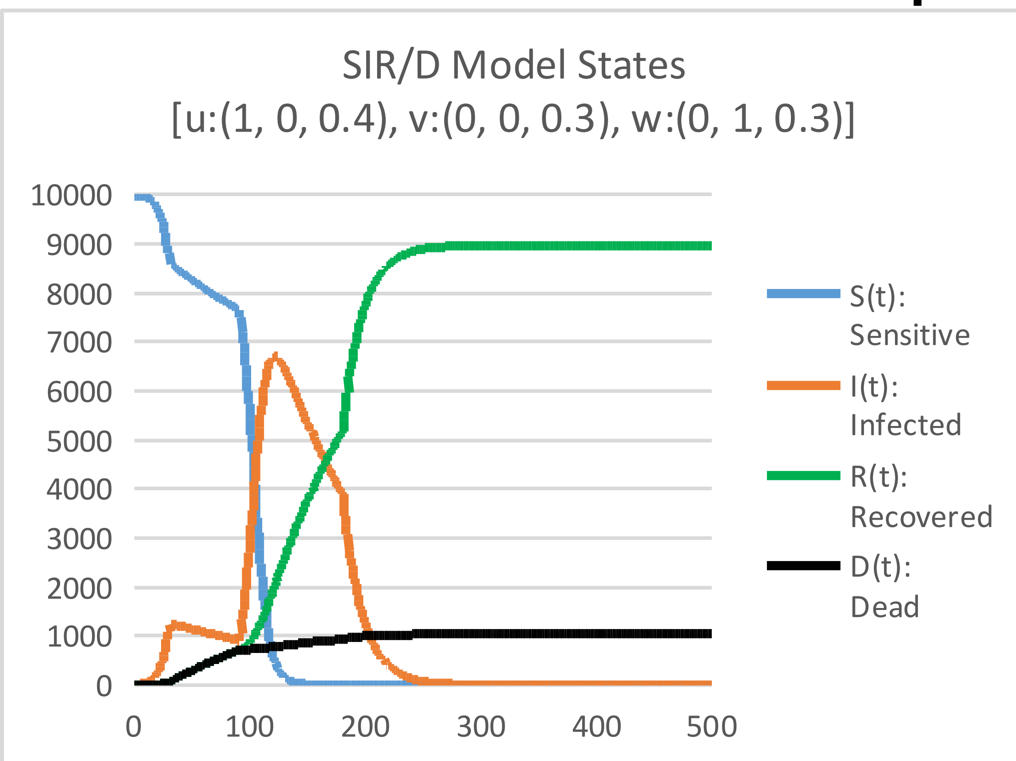
To extract knowledge from an epidemic control simulation.

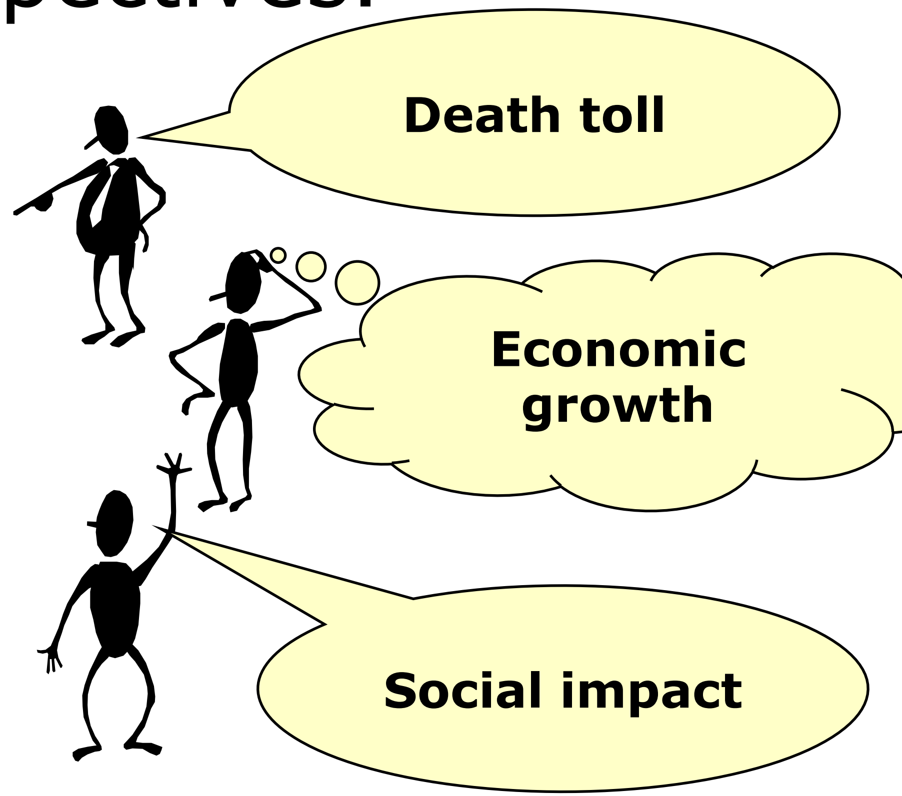
What knowledge?

- How simulation outcomes classified?
- What kind of cost-effective perspectives characterize those classification?
- Which outcomes are good or bad under what kind of cost-effective perspectives?

Background

- Epidemics affect society & economy.
- Cost-effectiveness may include various social & economic perspectives.





Approach

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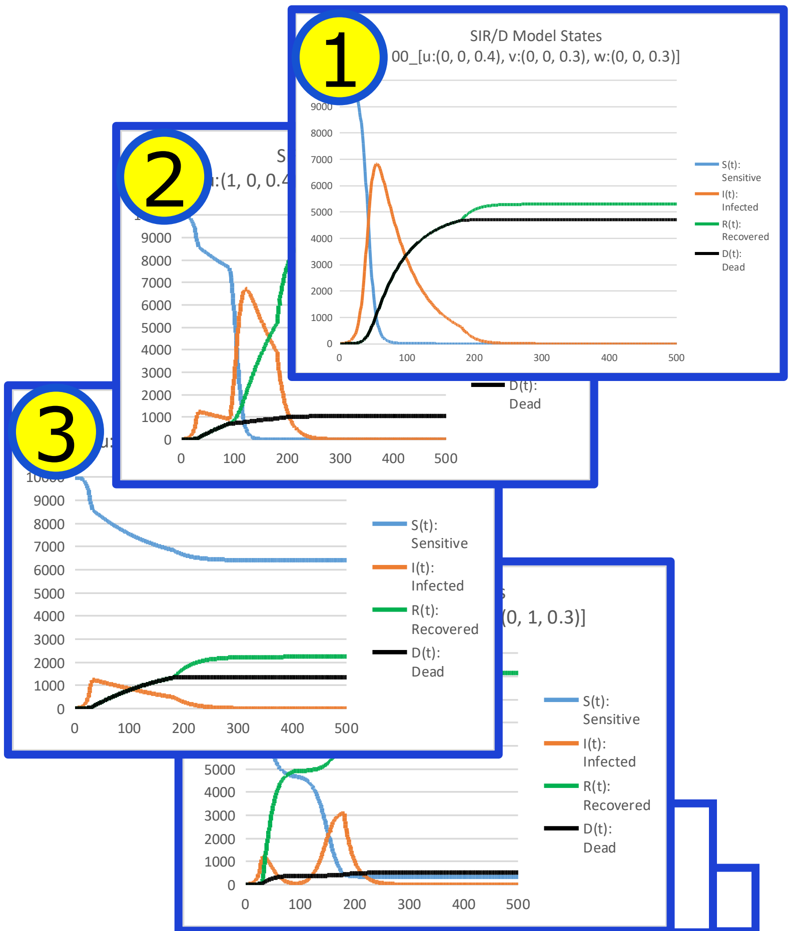
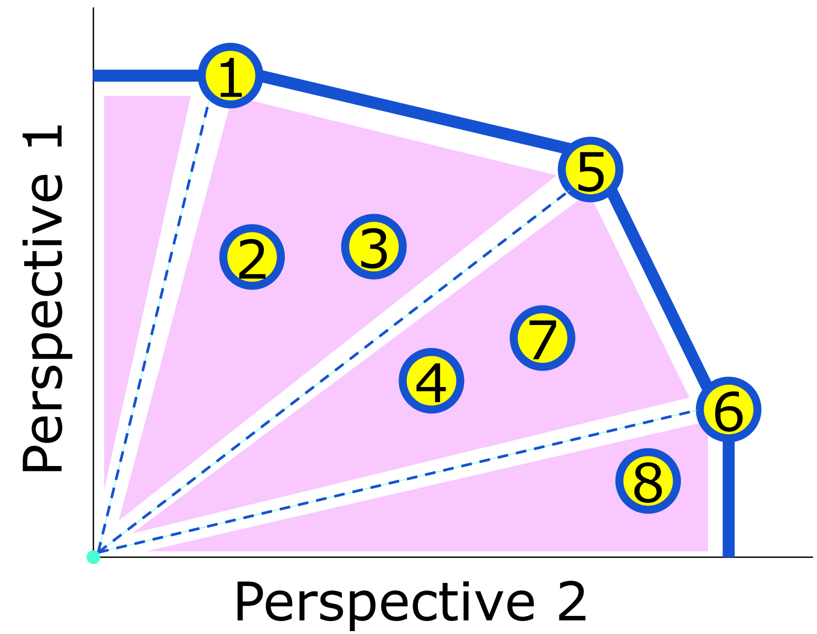
Data envelopment analysis (DEA) for simulation outcome classification.



Classified groups by weighted perspectives



Data envelopment analysis (DEA)

- Comparing efficiency of Decision Making Unit (DMU) with multiple input & out put. [CCR 1978]

- DMU efficiency is characterized by a reference set consisting of DMUs with optimal efficiency.

- Envelop (connecting reference sets) enables us to comparison of relative positions of DMUs.

DMU(Decision Making Unit)

DMU$_k$ ($k = 1,\sim,K$): M input ($\mathbf{x}$), N output ($\mathbf{y}$) system

$$\begin{pmatrix} x_{k1} \\ x_{k2} \\ \vdots \\ x_{kM} \end{pmatrix} \Rightarrow \boxed{\text{DMU}_k} \Rightarrow \begin{pmatrix} y_{k1} \\ y_{k2} \\ \vdots \\ y_{Nk} \end{pmatrix}$$

$m = 1\sim M$, $x_{km} > 0 \ ({}^{\forall}k, {}^{\forall}m)$

$k = 1\sim K$

$n = 1\sim N$, $y_{kn} > 0 \ ({}^{\forall}k, {}^{\forall}n)$

Efficiency Θ_k is given by wight vectors $\boldsymbol{\eta}_k$, $\boldsymbol{\xi}_k$

$$\theta_k = \sum_{n=1}^{N} \eta_{kn} y_{kn} \Big/ \sum_{m=1}^{M} \xi_{km} x_{km}$$

How to define η, ξ ?

How to define weight $\boldsymbol{\eta}$, ξ ?

- Optimize weight $\boldsymbol{\eta}_k$, $\boldsymbol{\xi}_k$ each for each DMU$_k$.
- When optimizing $\boldsymbol{\eta}_k$, $\boldsymbol{\xi}_k$ of DMU$_k$, weight $\boldsymbol{\eta}_k$, $\boldsymbol{\xi}_k$ are not allowed other DMU$_h$'s efficiency $\Theta_h > 1$.

$$\max_{\eta,\xi} \ \Theta_k = \sum_{n=1}^{N} \eta_n y_{kn} \Big/ \sum_{m=1}^{M} \xi_m x_{km}$$

Efficiency of DMU$_k$: Θ_k

$$\text{s.t.} \begin{cases} \sum_{n=1}^{N} \eta_n y_{hn} \Big/ \sum_{m=1}^{M} \xi_m x_{hm} \le 1 \ \left(h = 1 \sim K\right) \\ \eta_n \ge 0 \quad \left(n = 1 \sim N\right) \\ \xi_m \ge 0 \quad \left(m = 1 \sim M\right) \end{cases}$$

Restrict $\boldsymbol{\eta}, \xi$ to $\Theta_h \le 1$ for all other DMU$_h$

All weight not negative

$$\Rightarrow \xi_{kn} = \xi_n^*, \eta_{kn} = \eta_n^*$$

Reference set

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If optimal $\Theta^*_k < 1$, there exist DMU_h that obstacle $\Theta^*_k = 1$.

$$\max_{\eta_k, \xi_k} \theta_k = \sum_{n=1}^{N} \eta_{kn} y_{kn} \Big/ \sum_{m=1}^{M} \xi_{km} x_{km}$$

$$\text{s.t.} \begin{cases} \boxed{\sum_{n=1}^{N} \eta_{kn} y_{hn} \Big/ \sum_{m=1}^{M} \xi_{km} x_{hm} \le 1 \quad (h = 1 \sim K)} \\ \eta_{kn} \ge 0 \quad (n = 1 \sim N) \\ \xi_{km} \ge 0 \quad (m = 1 \sim M) \end{cases}$$

Such the h^{th}-DMU is efficient more than k^{th}-DMU.

Reference set of DMU_k : E_k

$$E_k = \left\{ h \,\middle|\, \sum_{n=1}^{N} \eta_{kn} y_{hn} \Big/ \sum_{m=1}^{M} \xi_{km} x_{hm} = 1 \ (h = 1 \sim K) \right\}$$

E_k works as target of improvement for k.

Classified by DEA

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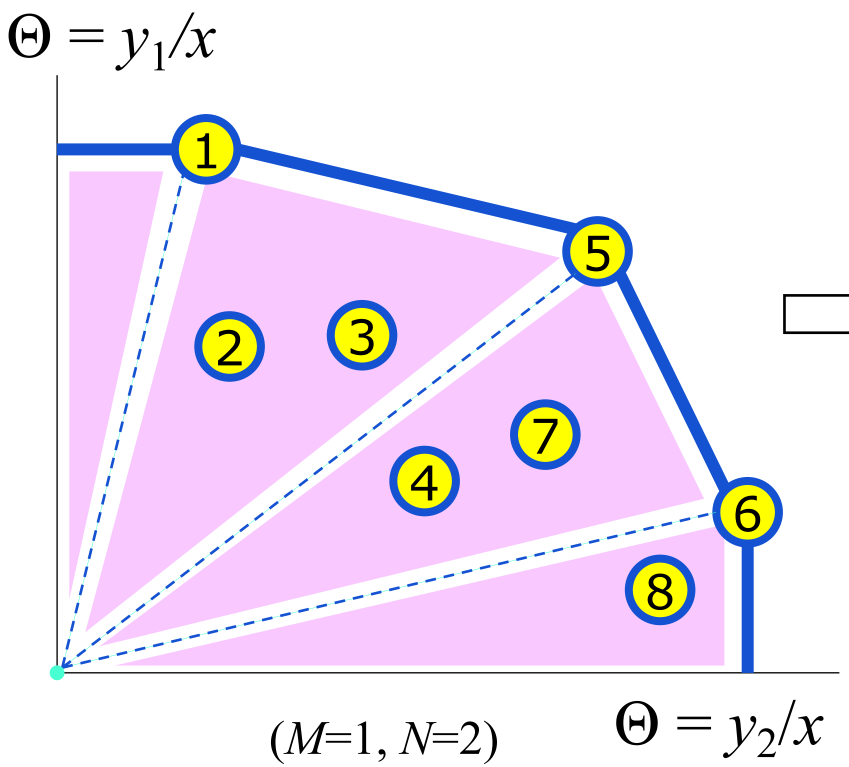
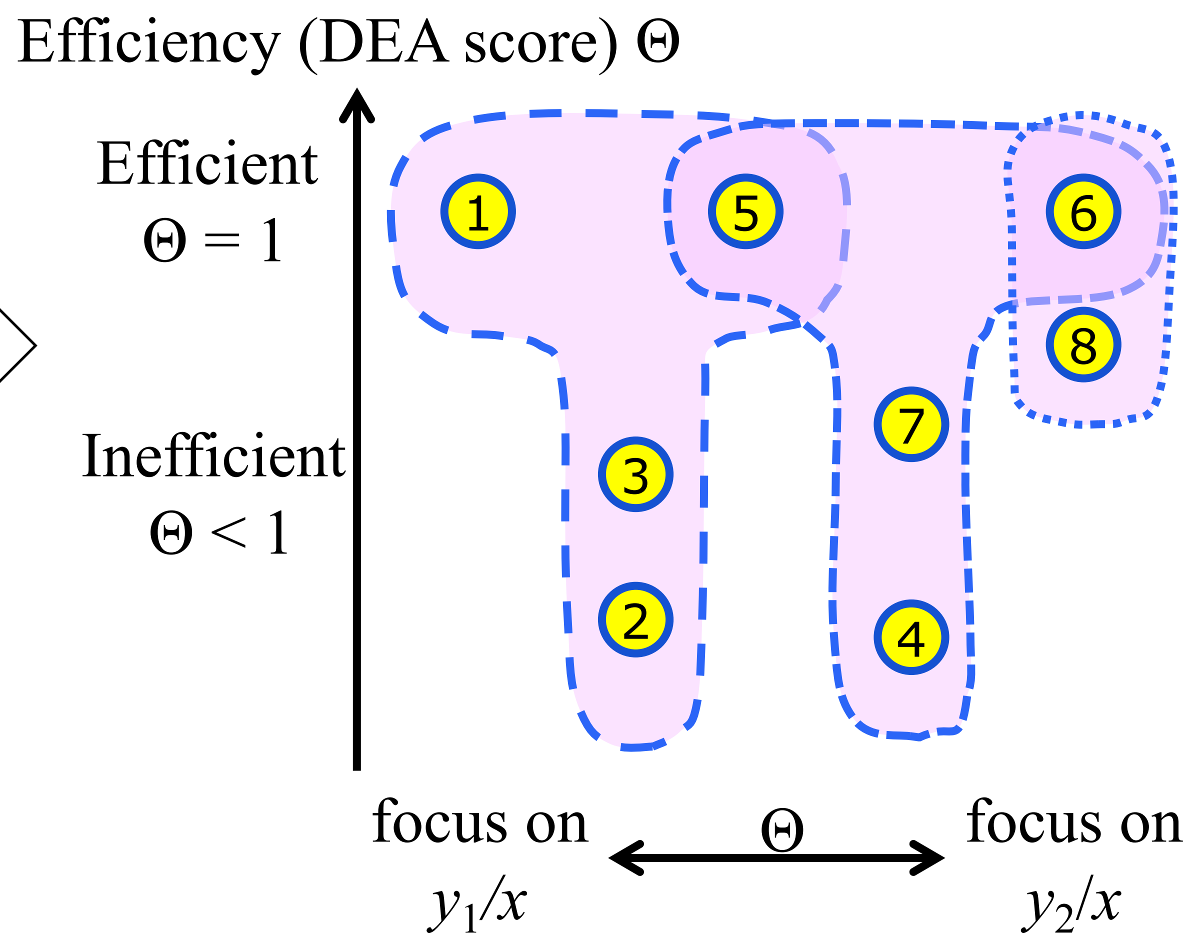
DMUs are divided into groups that has common reference set.





Epidemic Model

Kermack-McKendrick(1932)

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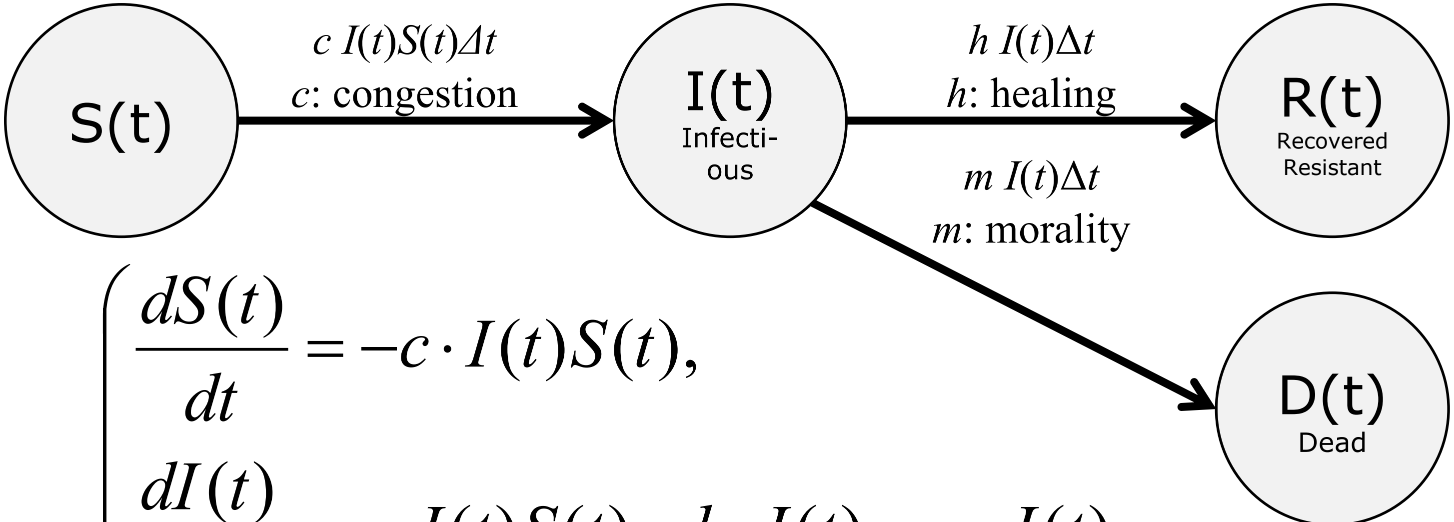

$$
\begin{cases}
\dfrac{dS(t)}{dt} = -c \cdot I(t)S(t), \\
\dfrac{dI(t)}{dt} = c \cdot I(t)S(t) - h \cdot I(t) - m \cdot I(t), \\
\dfrac{dR(t)}{dt} = h \cdot I(t), \\
\dfrac{dD(t)}{dt} = m \cdot I(t).
\end{cases}
$$

$$S(0) = P_0 - I(0),\ \ I(0) = \varepsilon,\ \ R(0) = 0,\ \ D(0) = 0.$$

c : contagion, h : heeling,, m : mortality,

Epidemic Control Model

$$\begin{cases} \dfrac{dS(t)}{dt} = -c(t)S(t)I(t), \ \ S(0) = P_0 - \varepsilon, \ \ c(t): \text{contagion}, \\ \dfrac{dI(t)}{dt} = c(t)S(t)I(t) - h(t)I(t) - m(t)I(t), \ \ I(0) = \varepsilon, \\ \dfrac{dR(t)}{dt} = h(t)I(t), \ \ R(0) = 0, \ \ h(t): \text{heeling}, \\ \dfrac{dD(t)}{dt} = m(t)I(t), \ \ D(0) = 0, \ \ m(t)\text{:mortality} \\ P(t) = S(t) + I(t) + R(t), \quad \text{population alive} \\ P(0) = P_0, \quad \text{initial population.} \end{cases}$$

$$\begin{cases} c(t) = \dfrac{c_0}{1 + \alpha_u \cdot u(t)}, \\ h(t) = h_0 (1 + \alpha_v \cdot v(t)), \\ m(t) = \dfrac{m_0}{1 + \alpha_w \cdot w(t)}. \end{cases}$$

$$\begin{cases} u(t), v(t), w(t) \geq 0 \\ u(t) + v(t) + w(t) \leq 1 \end{cases}$$

$u(t)$: Contagion Suppression

$v(t)$: Recovery Promotion

$w(t)$: Mortality Suppression

Coefficients and Initial Conditions

Common Simulation Setting				
Initial Conditions	$S(0)$: Susceptibles	$I(0) = \varepsilon$: Infectives	$R(0)$: Recovered	$D(0)$: Dead
	9990	10	0	0
	$P(0)$: Population Alive =10000			
Coefficients	c_0 Contagion	h_0 Healing	m_0 Mortality	
	0.00002	0.01	0.01	
Control Sensitivities	α_u	α_v	α_w	
	10	15	20	

Control Strategies for Simulation Run

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- $u(t)$: Contagion Suppression
- $v(t)$: Recovery Promotion
- $w(t)$: Mortality Suppression

	Stage-0 t=0~29			Stage-1 t=30~89			Stage-2 t=90~179			Stage-3 t=180~500		
RUN # (DMU #)	$u(t)$	$v(t)$	$w(t)$	$u(t)$	$v(t)$	$w(t)$	$u(t)$	$v(t)$	$w(t)$	$u(t)$	$v(t)$	$w(t)$
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.3	0.3
1				1.0	0.0	0.0	1.0	0.0	0.0			
2				1.0	0.0	0.0	0.0	1.0	0.0			
3				1.0	0.0	0.0	0.0	0.0	1.0			
4				0.0	1.0	0.0	1.0	0.0	0.0			
5				0.0	1.0	0.0	0.0	1.0	0.0			
6				0.0	1.0	0.0	0.0	0.0	1.0			
7				0.0	0.0	1.0	1.0	0.0	0.0			
8				0.0	0.0	1.0	0.0	1.0	0.0			
9				0.0	0.0	1.0	0.0	0.0	1.0			

Baseline (minimum control) case

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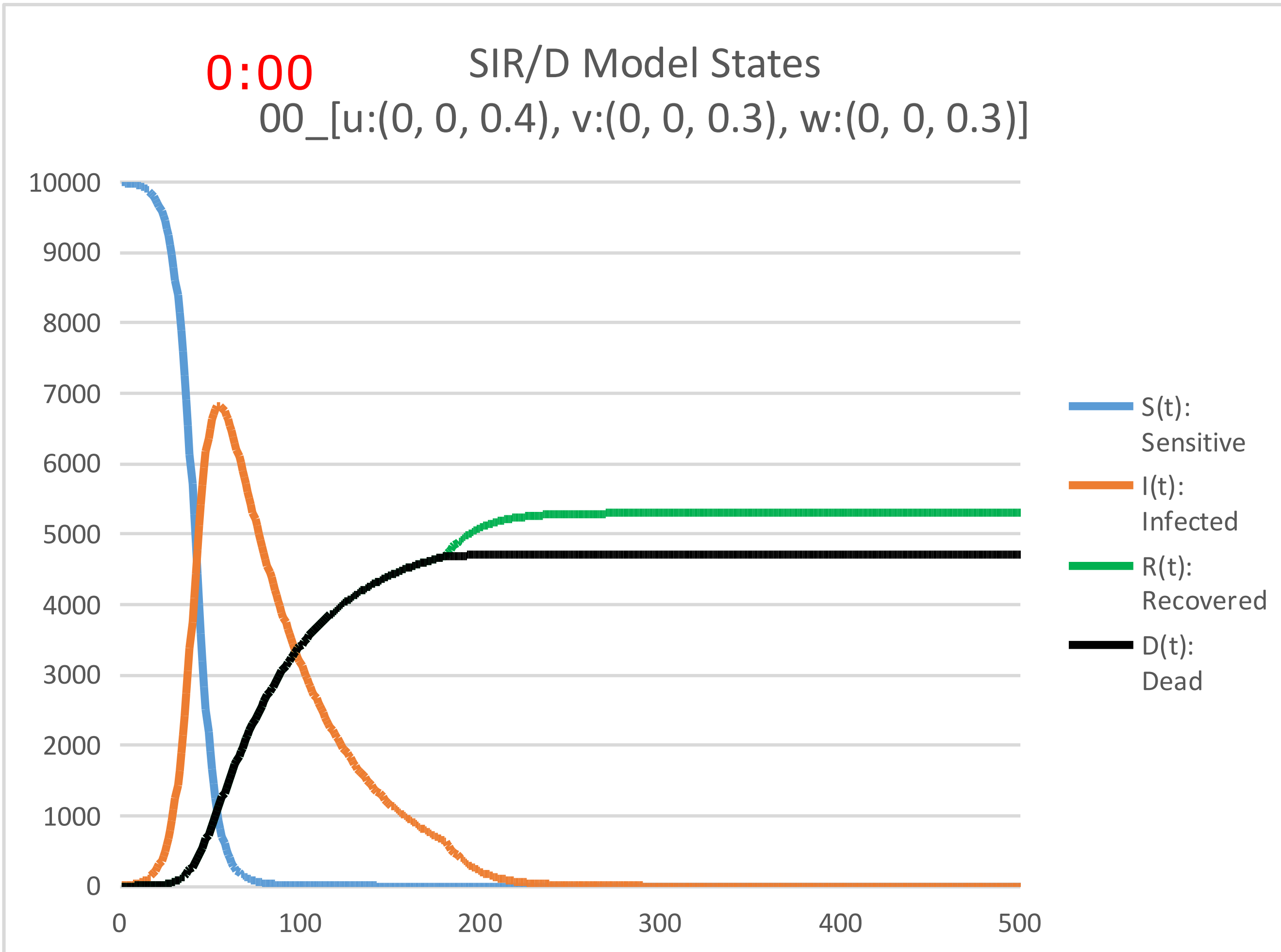
Coeff.: c_0=0.00002, h_0=0.01, m_0=0.01,
Initial Conditions: S(0)=9990, I(0)=10, R(0)=-0, D(0)=0, Sensitivities: a_u=10, a_v=15, a_w=20,
Control Stages: 0th:0-29, 1st:30-89, 2nd:90-179, 3d:180-



Other Simulation Runs

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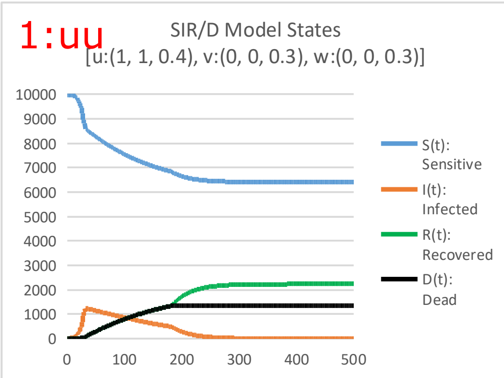

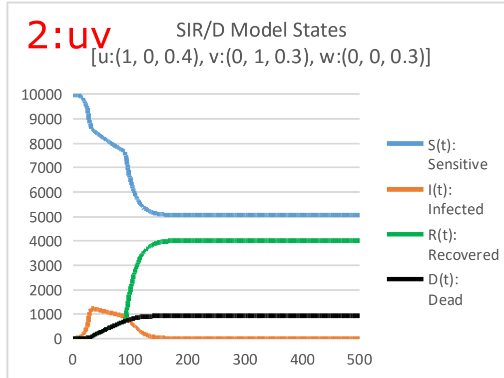

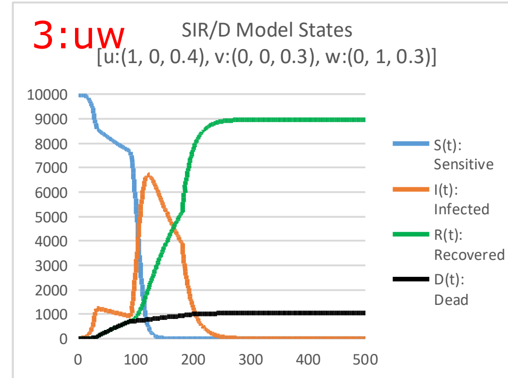

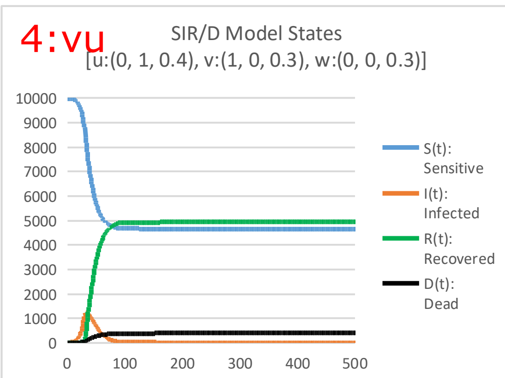

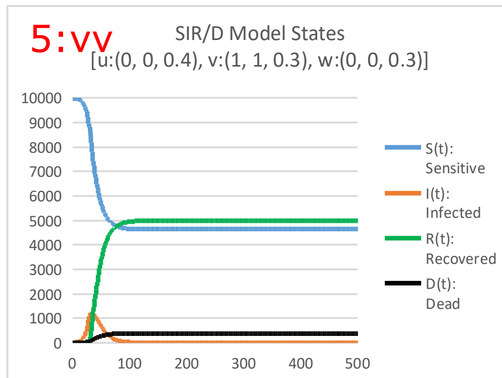

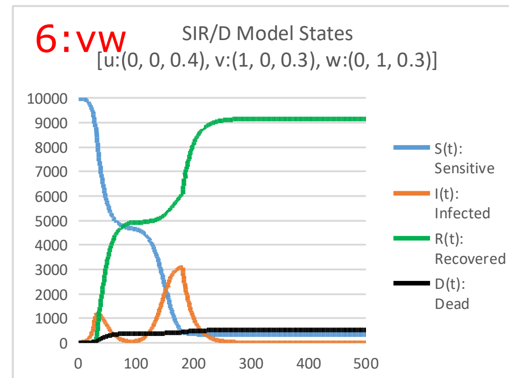

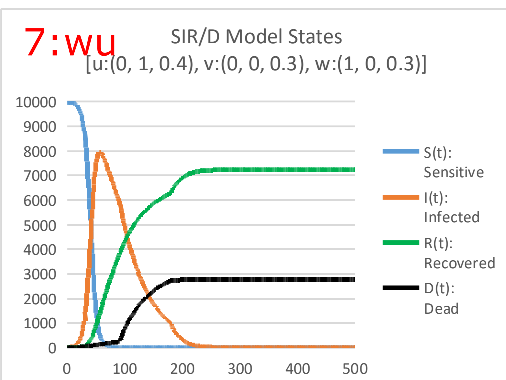

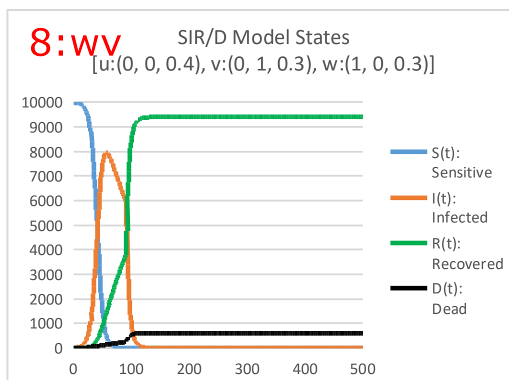

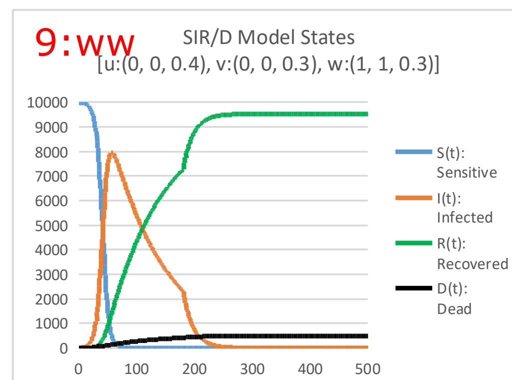

Perspectives for evaluation

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Input (**x**)	Time average of total cost for contagion suppression $Cs = \mathrm{Average}[P(t)\cdot\alpha_u\cdot u(t)]$
	Time average of total remedial cost $Cm = \mathrm{Average}[I(t)\cdot\{\alpha_v\cdot v(t) + \alpha_w\cdot w(t)\}]$
Output (**y**)	Bottom of uninfected or recovered population (Impact to social & economicl activities) $BtmActP = \mathrm{Min}[P(t)\text{-}I(t)]$
	Time average of uninfected or recovered population (Accumulated influence to social & economicl activities) $AvgActP = \mathrm{Average}[P(t)\text{-}I(t)]$
	Survived population (Avoided number of death) $AlvPE = \mathrm{Min}\ P(t)$

Summary of input(x) and output(y)

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		Simulation Result (Summary Values)				
		***Input* (x):**		***Output* (y):**		
		Cs	Cm	BtmActP	AvgActP	AlvPE
		Social Cost:	Medical Cost:	Bottom Active Population:	Average Active Population:	Alive Population at the end:
		$Avg[P(t) \cdot \alpha_u \cdot u(t)]$	$Avg[I(t)\{\alpha_v \cdot v(t) + \alpha_w \cdot w(t)\}]$	Min[P(t)-I(t)]	Avg[P(t)-I(t)]	Min[P(t)]
Run # (DMU #)	0	13,574	231	1,915	5,112	5,296
	1	49,693	340	8,176	8,590	8,638
	2	34,737	617	8,371	9,011	9,072
	3	34,481	19,220	2,514	7,974	8,967
	4	41,879	912	8,650	9,560	9,603
	5	24,652	919	8,650	9,575	9,619
	6	24,328	6,776	6,505	9,147	9,488
	7	33,203	15,244	1,906	6,589	7,235
	8	24,088	15,932	1,906	8,663	9,399
	9	24,418	29,615	1,906	8,082	9,524

DEA Outcome

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DEA_CCR		DEA Score Θ	Reference Set	Input Weights ξ		Output Weights η		
				ξ_Cs	ξ_Cm	$\eta\_BtmActP$	$\eta\_AvgActP$	η_AlvPE
Run # (DMU #)	0	1.0	{0, 1, 2}	0.4	22.4	1.8	0	1.2
	1	1.0	{1}	0	29.4	1.2	0	0
	2	1.0	{1, 2}	0.1	8.4	1.2	0	0
	3	0.6665	{8, 5}	0.3	0	0	0	0.7
	4	0.7903	{0, 2, 5}	0.1	5.3	0.5	0.4	0
	5	1.0	{5}	0.4	0	1.2	0	0
	6	0.9995	{8, 5}	0.4	0	0	0	1.1
	7	0.5584	{8, 5}	0.3	0	0	0	0.8
	8	1.0	{8}	0.4	0	0	0	1.1
	9	0.9997	{8}	0.4	0	0	0	1.0

DEA classification

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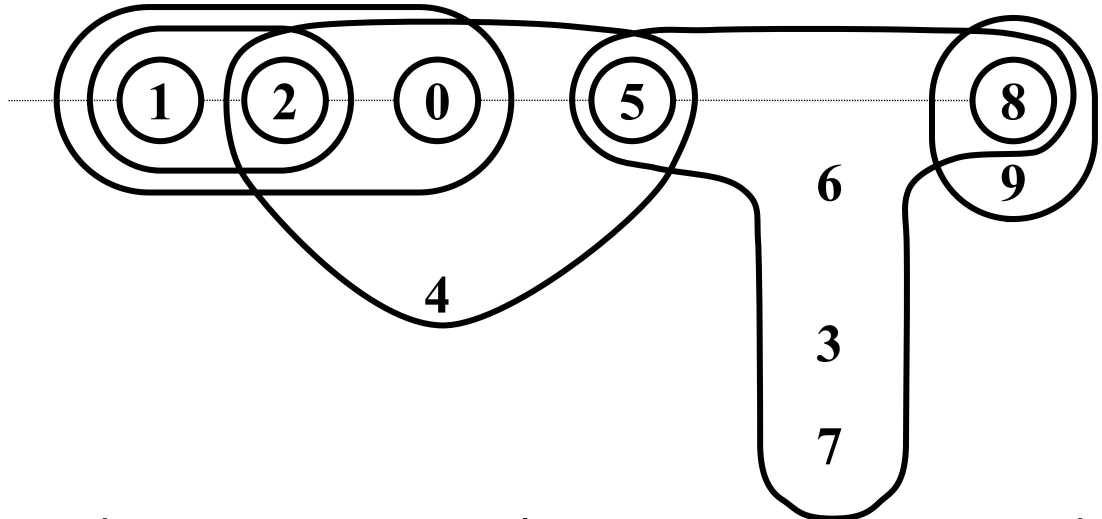
Reference Sets
(Efficient: $\Theta=1$)

↕

Inefficient: $\Theta<1$



Characteristics on evaluative weight
ξ , η

$\xi_{Cm} > 5.0$ $\qquad$ $\xi_{Cm} = 0$

$\eta_{BtmActP} > 0.5$ $\qquad$ $\eta_{BtmActP} = 0$

$\xi_{Cs} > 0$ & $\eta_{AlvPE} > 0$,
others = 0

Focused efficiency
Θ

$$\frac{\text{Bottom of activities (BtmActP)}}{\text{Remedial cost (Cm)}} \qquad \frac{\text{Avoided \# of death (AlvPE)}}{\text{Contagion suppression cost (Cs)}}$$

Knowledge extraction

What knowledge we wanted?

1. How simulation outcomes classified?

2. What kind of cost-effective perspectives characterize those classification?

3. Which outcomes are good or bad under what kind of cost-effective perspectives?

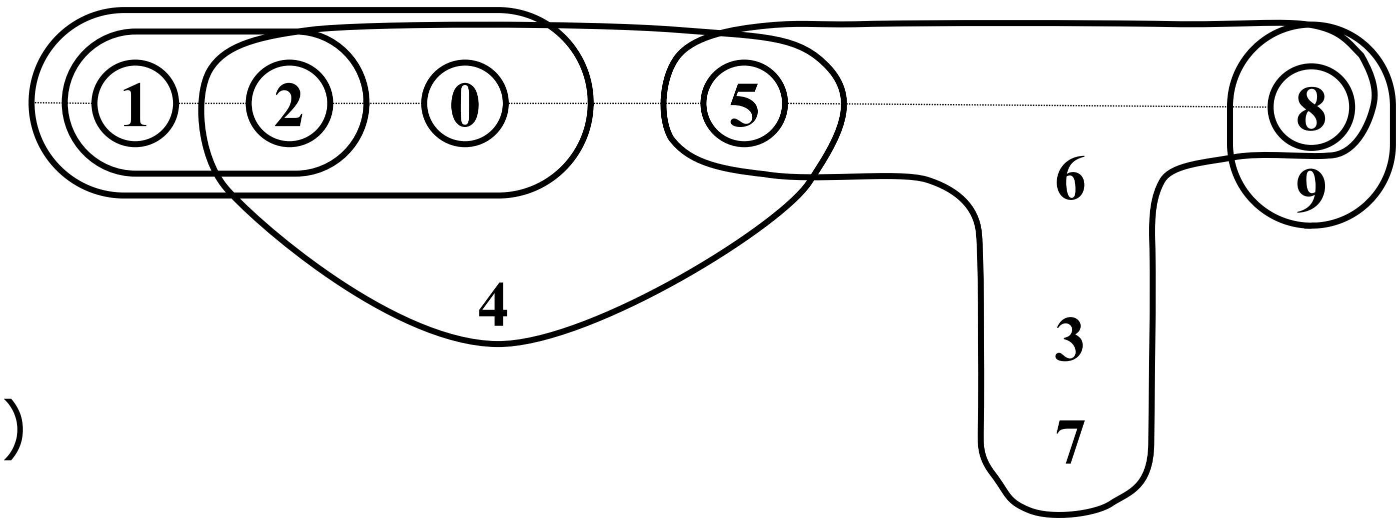
Extracted knowledge by DEA

1. Classification of epidemic control policies 0 ~ 9

3. Good (efficient)

↕

Bad (Inefficient)



2. Cost-effective perspectives characterizing the groups

Focused efficiency

$$\frac{\text{Bottom of activities}}{\text{Remedial cost}} \qquad \frac{\text{Avoided \# of death}}{\text{Contagion suppression cost}}$$

Summary

- Purpose:
  Extract knowledge from epidemic control simulation
- Results:
  - Classified epidemic control policies outcome,
  - Found perspectives of cost-effectiveness characterize those classification,
  - Found which control policies are good or bad under those perspective.