# Ambiguity and vagueness of fuzzy number data and fuzzy measure weight for AHP

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# Introduction

Analytic Hierarchy Process (AHP) methodology is a very convenient and popular in the multi criterion decision making field.

- ✓ Data matrix must have enough consistency for its reliability
- ✓ Elements of levels must be independent perfectly.

HOWEVER, in practice, it is very difficult

- ➢ Enough consistency in the data reciprocal matrix.
- ➢ Perfect independence among elements (criteria and alternatives).

Extended methods

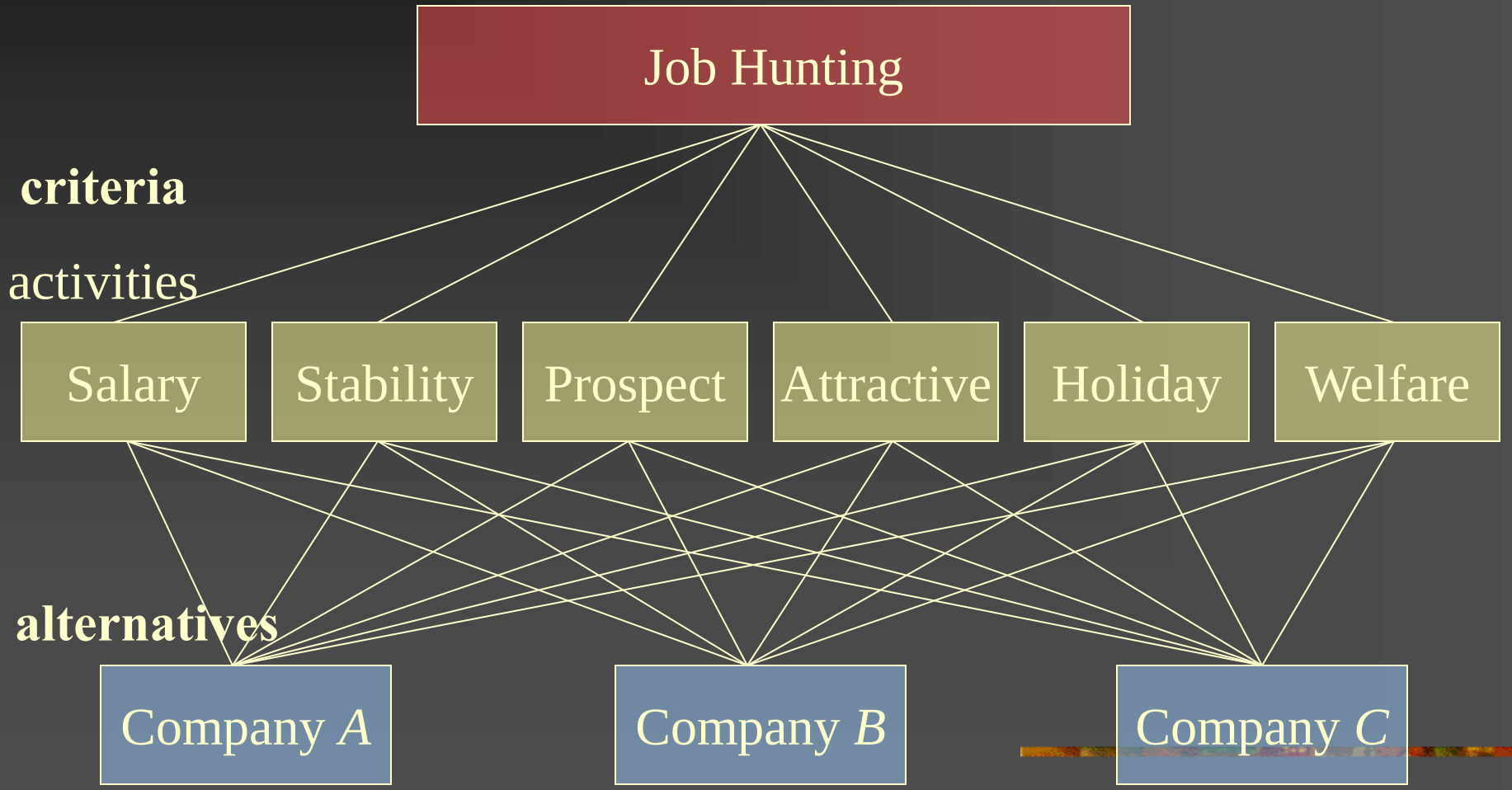
- ■ Fuzzy number data AHP
- ■ Fuzzy measure weight AHP

**In this study**

- ◆ **Comparative consideration between these two methods from a view point of their "Fuzziness" (ambiguity and vagueness)**

1. **Representation by a hierarchy**
2. Pairwise comparison matrices
3. (Consistency check)
4. Local weights of criteria
5. Global weights of alternative

# Hierarchy structure



# Fuzzy number data AHP

## reciprocal matrix (Ohnishi et al. 2006)

- An extension of the Normal AHP for inconsistency or unreliability of data
- Employing fuzzy number (fuzzy set) as components of pair-wise data
- Using fuzzy number calculation for reciprocity of data matrix
- Adopting membership value of fuzzy set instead of consistency index for data reliability

# Fuzzy reciprocal data matrix

Reciprocity $\mu_{ij}(r) = \mu_{ji}(1/r)$

$$\text{core}(\tilde{r}_{ji}) = 1/r_{ij}$$
$$\text{supp}(\tilde{r}_{ji}) = [1/u_{ij}, 1/l_{ij}]$$

Assumption

if $r_{ij} \geq 1 \rightarrow \tilde{r}_{ij}$ is a triangular fuzzy number

$\tilde{r}_{ij} = (l_{ij}, r_{ij}, u_{ij})_{\Delta}$

( $\tilde{r}_{ji}$ may not be a triangular )

else $r_{ij} < 1 \rightarrow \tilde{r}_{ji} = (1/u_{ij}, 1/r_{ij}, 1/l_{ij})_{\Delta}$

# Optimal degree of satisfaction and weight

$$\alpha^* \equiv \max_{w_1,\ldots,w_n} \min_{i,j} \left\{ \mu_{ij}\left( \frac{w_i}{w_j} \right) \right\}$$

If all $\tilde{r}_{ij}$ $(i < j)$ are triangular fuzzy numbers $(l_{ij}, r_{ij}, u_{ij})_{\Delta}$,

**[NLP]**

Maximize $\alpha$

$$w_j \{ l_{ij} + \alpha(r_{ij} - l_{ij}) \} \le w_i \le w_j \{ u_{ij} + \alpha(r_{ij} - u_{ij}) \}$$

$$\sum_{i}^{n} w_i = 1 \qquad (i, j = 1,\ldots,n)$$

# Fuzzy Measure weight AHP (Ichihashi 1989)

- An extension of the Normal AHP for dependency structure
- Using fuzzy measure as non-additive weight
- Employing Choquet integral for aggregating total priority.
- Two types of decision by use of non-additive fuzzy measure
  - Substitutive decision (possibility measure)
  - Complementary decision (necessity measure)

# Overall weights of fuzzy measure AHP

<Substitutive decision>

The upper limit expectation based on possibility measure

$$y_p^{(\mathrm{Pl})} = \sum_{l}^{q} m(A_l) \max_{x_i \in A_l} f_p(x_i)$$

<Complementary decision>

The lower limit expectation based on necessity measure

$$y_p^{(\mathrm{Bel})} = \sum_{l}^{q} m(A_l) \min_{x_i \in A_l} f_p(x_i)$$

$f_p(x_i)$ : weights of $p$-th alternative with respect to $x_i$

# Fuzzy theory

## Fuzzy Set

「old vase」

This pot *x*: 100 years old (crisp)

Set *A*: old vase:
not determined how old (fuzzy)

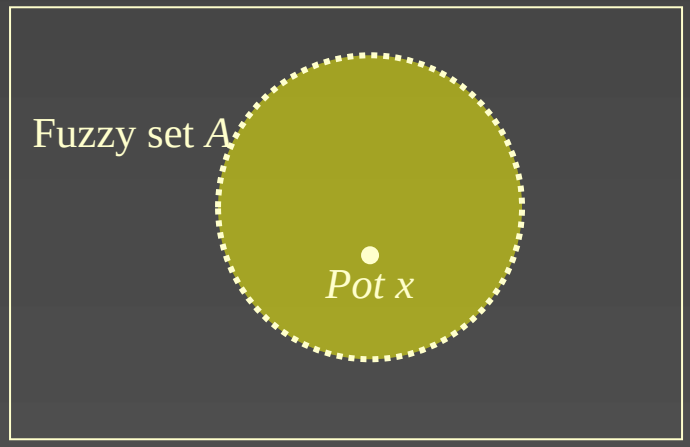
## Fuzzy Measure

「old vase」

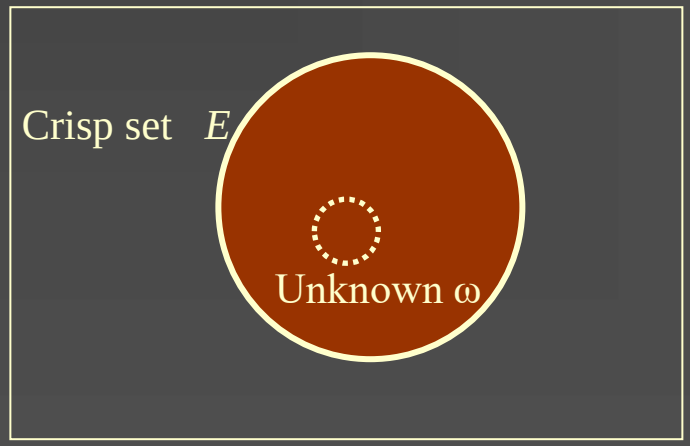
This pot ω: unknow how old (fuzzy)

Set *E*: old vase:
over 100 years old (crisp)

## Vagueness




## Ambigyuity



# Fuzzy number data AHP

- Fuzziness to resolve is “vagueness”, because it employs fuzzy set as data value.
- “ambiguity” may remain
- It is easy for decision maker to make pair-wise data matrix, because it does not take exact value.
- It is difficult to understand reciprocity and inconsistency of data matrix.

# Fuzzy measure weight AHP

- Fuzziness to resolve is “ambiguity” because it employs non-additive fuzzy measure as weight of criteria.
- “vagueness” may remain
- It is difficult for decision maker to understand ideas
  - ✓ Weights of subsets but not of each criterion (elements)
  - ✓ Fuzzy integral as aggregation.

# Conclusions

- ✓ Fuzzy reciprocal number data AHP
- ✓ Fuzzy measure weight AHP
- ◆ “Fuzziness” (ambiguity OR vagueness) may remain in the two extension methods.
- ◆ There are some weak points and strong points in each methods.
- ◆ We can select different type of cost in each methods.

## In the future

- ◆ we will combine the two methods for eliminate ambiguity AND vagueness.