Achievement of Collision Avoidance and Formation for Nonlinear Multi-Ship Systems Using An Interval Type-2 Fuzzy Tracking Approach

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Problem Statements
- Multi-Autonomous Ships

Application of Multi-Autonomous Ships:

In existing papers on multi-agent control systems, multi-ship systems have been shown to provide better efficiency in completing various tasks.

Exploration & Monitoring

Fig. P1. Environmental monitor [P1]

Environmental Issue (Clean-up)

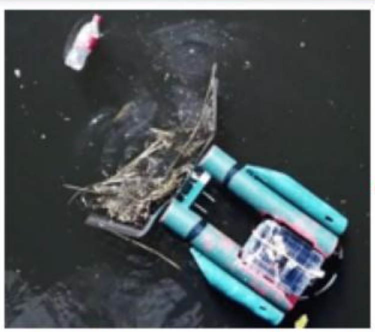

Fig. P2. Water surface cleaner [P2]

Multi-Ships (Multi-Agent System)

Fig. P3. Application by Using Multiple Ships [P3]

[P1] https://en.hi-target.com.cn/iboat-bs3-usv
[P2] https://newatlas.com/chicago-river-trash-robot/53864/
[P3] https://www.oceanalpha.com/application_cases/choosing-the-right-usv-5-considerations-to-get-you-started/

Problem Statements
- Control Problems

Formation and Collision Avoidance Control Problems:

In this research, the formation and collision avoidance control problems of multi-ship systems have been discussed with four leader ships and two obstacles.

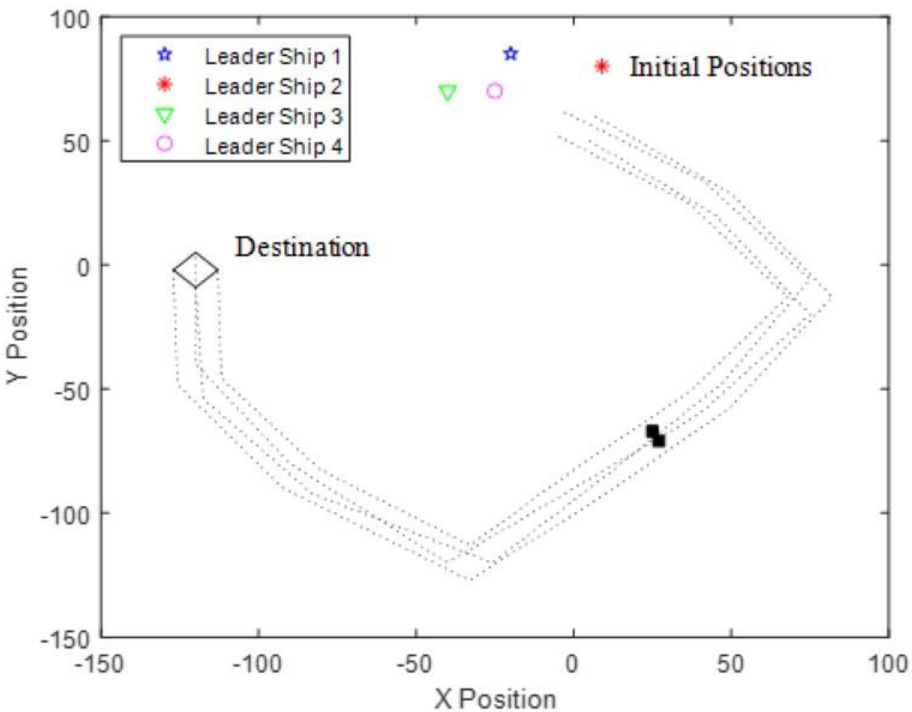

Figure. 1: Formation and collision avoidance problems.

Formation Control Problem

Formation control method arranges agents into a specified formation with relative positions.

Multi-Ship Systems :

To place and maintain four leader ships in a rectangular pattern until arrival at the destination.

Collision Avoidance Problem

Collision avoidance control changes the course to stay clear of obstacles or other agents.

Multi-Ship Systems :

To keep four leader ships safe from two obstacles and each other during formation and navigation.

System Description
- Ship Dynamic System

Nonlinear Multi-Ship Systems:

Extending the nonlinear ship dynamic system in [19] to the control problem of multi-autonomous ship systems, the dynamic equations can be presented as follows for $\varepsilon = 1, 2, 3, 4$.

$$\dot{x}_1^{\varepsilon}(t) = \left(cos\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_4^{\varepsilon}(t) - \left(sin\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_5^{\varepsilon}(t)$$
$$\dot{x}_2^{\varepsilon}(t) = \left(sin\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_4^{\varepsilon}(t) + \left(cos\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_5^{\varepsilon}(t)$$
$$\dot{x}_3^{\varepsilon}(t) = \left(1 + \Delta^{\varepsilon}(t)\right)x_6^{\varepsilon}(t)$$
$$\dot{x}_4^{\varepsilon}(t) = -0.0318x_4^{\varepsilon}(t) + 0.8870u_1^{\varepsilon}(t)$$
$$\dot{x}_5^{\varepsilon}(t) = -0.0628x_5^{\varepsilon}(t) - 0.0030x_6^{\varepsilon}(t) + 0.5415u_2^{\varepsilon}(t) + 0.3152u_3^{\varepsilon}(t)$$
$$\dot{x}_6^{\varepsilon}(t) = -0.0045x_5^{\varepsilon}(t) - 0.2427x_6^{\varepsilon}(t) + 0.3152u_2^{\varepsilon}(t) + 8.0082u_3^{\varepsilon}(t)$$

Describe

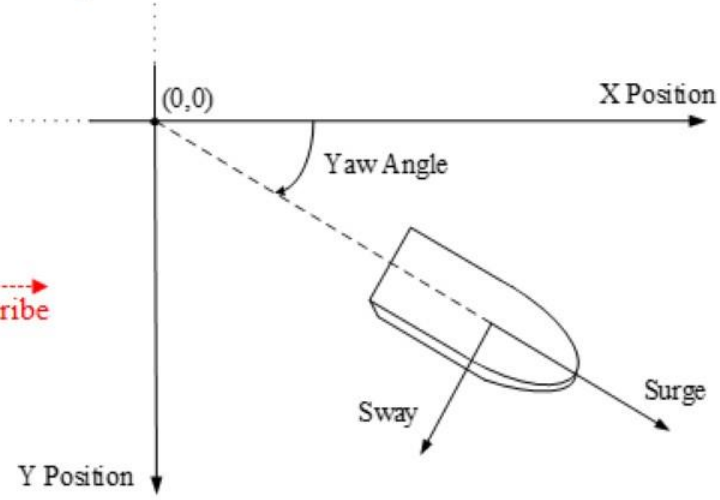

Fig. P4. States of nonlinear ship dynamic system

★ *System Variables:*

$x_1^{\varepsilon}(t)$: X Position $x_3^{\varepsilon}(t)$: Yaw Angle $x_5^{\varepsilon}(t)$: Sway Motion

$x_2^{\varepsilon}(t)$: Y Position $x_4^{\varepsilon}(t)$: Surge Motion $x_6^{\varepsilon}(t)$: Yaw Angular Velocity

$$\begin{cases} \Delta^{1,2}(t) = 0.1cos(t) \\ \Delta^{3,4}(t) = 0.1sin(t) \end{cases}$$: Uncertainties

[19] T. I. Fossen, and A. Grovlen, "Nonlinear Output Feedback Control of Dynamically Positioned Ships Using Vectorial Observer Backstepping," IEEE Transactions on Control Systems Technology, vol. 6, pp. 121-128, Jan. 1998, doi:10.1109/87.654882.

System Description
- IT-2 T-S Fuzzy Model

The Overall IT-2 T-S Fuzzy Model:

Following the same modeling concept as in [16] for nonlinear multi-ship systems, the overall T-SFM is constructed as follows.

Nonlinear Multi-Ship Dynamic Systems

$$\dot{x}_1^{\varepsilon}(t) = \left(\cos\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_4^{\varepsilon}(t) - \left(\sin\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_5^{\varepsilon}(t)$$

$$\dot{x}_2^{\varepsilon}(t) = \left(\sin\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_4^{\varepsilon}(t) + \left(\cos\left(x_3^{\varepsilon}(t)\right) + \Delta^{\varepsilon}(t)\right)x_5^{\varepsilon}(t)$$

$$\dot{x}_3^{\varepsilon}(t) = \left(1 + \Delta^{\varepsilon}(t)\right)x_6^{\varepsilon}(t)$$

$$\dot{x}_4^{\varepsilon}(t) = -0.0318x_4^{\varepsilon}(t) + 0.8870u_1^{\varepsilon}(t)$$

$$\dot{x}_5^{\varepsilon}(t) = -0.0628x_5^{\varepsilon}(t) - 0.0030x_6^{\varepsilon}(t) + 0.5415u_2^{\varepsilon}(t) + 0.3152u_3^{\varepsilon}(t)$$

$$\dot{x}_6^{\varepsilon}(t) = -0.0045x_5^{\varepsilon}(t) - 0.2427x_6^{\varepsilon}(t) + 0.3152u_2^{\varepsilon}(t) + 8.0082u_3^{\varepsilon}(t)$$

Select

Operating Points

Point 1 $x_{(op1)}^{\varepsilon} = \begin{bmatrix} 0 & 0 & 90^{\circ} & 0 & 0 & 0 \end{bmatrix}^{T}$

Point 2 $x_{(op2)}^{\varepsilon} = \begin{bmatrix} 0 & 0 & 0^{\circ} & 0 & 0 & 0 \end{bmatrix}^{T}$

Point 3 $x_{(op3)}^{\varepsilon} = \begin{bmatrix} 0 & 0 & -90^{\circ} & 0 & 0 & 0 \end{bmatrix}^{T}$

Design

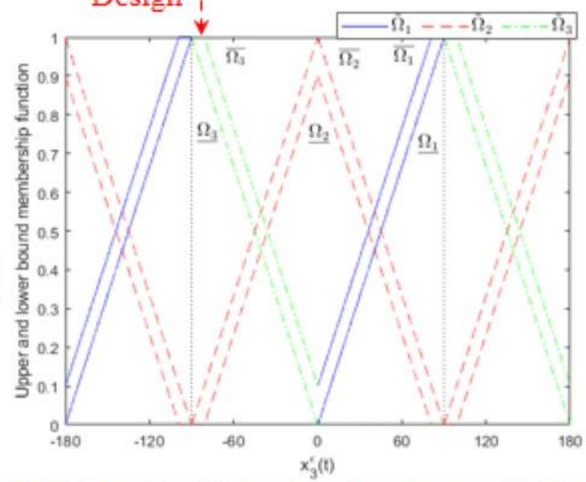

Figure 2. IT-2 membership function of nonlinear multi-ship systems

Construct

Represent

Overall IT-2 T-S Fuzzy Model

$$\dot{x}^{\varepsilon}(t) = \sum_{\alpha=1}^{3} \tilde{\Omega}_{\alpha}\left(x_3^{\varepsilon}(t)\right)\left\{\mathbf{A}_{\alpha}x^{\varepsilon}(t) + \mathbf{B}_{\alpha}u^{\varepsilon}(t)\right\}$$

where
- $x^{\varepsilon}(t) = \begin{bmatrix} x_1^{\varepsilon}(t) & x_2^{\varepsilon}(t) & x_3^{\varepsilon}(t) & x_4^{\varepsilon}(t) & x_5^{\varepsilon}(t) & x_6^{\varepsilon}(t) \end{bmatrix}^{T}$, $u^{\varepsilon}(t) = \begin{bmatrix} u_1^{\varepsilon}(t) & u_2^{\varepsilon}(t) & u_3^{\varepsilon}(t) \end{bmatrix}^{T}$
- $\alpha = 1, 2, 3$ are fuzzy rule numbers
- $\tilde{\Omega}_{\alpha}\left(x_3^{\varepsilon}(t)\right)$ are the IT-2 membership function

[16] W. J. Chang, Y. H. Lin, Y. C. Lee, and C. C. Ku, "Investigating Formation and Containment Problem for Nonlinear Multi-Agent Systems via Interval Type-2 Fuzzy Sliding Mode Tracking Approach," IEEE Transactions on Fuzzy Systems, Jul. 2024, 10.1109/TFUZZ.2024.3387045

System Description
- IT-2 T-S Fuzzy Model

IT-2 T-S Fuzzy Model of Tracking Purpose:

To achieve the tracking purposes of both formation and collision avoidance, the target model is also constructed and the tracking error IT-2 fuzzy model is obtained.

IT-2 Fuzzy Model for All Ships

$$ {}^{m}\dot{x}^{\varepsilon}(t)=\sum_{\alpha=1}^{3}\tilde{\Omega}_{\alpha}\left({}^{m}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{A}_{\alpha}\,{}^{m}x^{\varepsilon}(t)+\mathbf{B}_{\alpha}\,{}^{m}u^{\varepsilon}(t)\right\} $$

IT-2 Fuzzy Target Model

$$ {}^{m}\dot{x}_{d}^{\varepsilon}(t)=\sum_{\alpha=1}^{3}\tilde{\Omega}_{\alpha}\left({}^{m}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{A}_{\alpha}\,{}^{m}x_{d}^{\varepsilon}(t)\right\} $$

where ${}^{m}x_{d}^{\varepsilon}(t)$ is the desired state trajectory; $m=c,f$ denotes the collision avoidance mode and formation mode

$\varepsilon = 1,2,3,4$

Error Dynamic Model for Ships

$$ {}^{m}\dot{e}^{\varepsilon}(t)=\sum_{\alpha=1}^{3}\tilde{\Omega}_{\alpha}\left({}^{m}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{A}_{\alpha}\,{}^{m}e^{\varepsilon}(t)+\mathbf{B}_{\alpha}\,{}^{m}u^{\varepsilon}(t)\right\} $$

where ${}^{m}e^{\varepsilon}(t)={}^{m}x^{\varepsilon}(t)-{}^{m}x_{d}^{\varepsilon}(t)$

IT-2 Fuzzy Controller Design

- Imperfect Premise Matching

Imperfect Premise Matching Fuzzy Controller Design:

Applying the imperfect premise matching design method, the IT-2 fuzzy tracking controller can be constructed with different IT-2 membership function and rule number of the model.

Error Dynamic Model for Ships

$$ {}^{m}\dot{e}^{\varepsilon}(t)=\sum_{\alpha=1}^{3}\tilde{\Omega}_{\alpha}\left(x_3^{\varepsilon}(t)\right)\left\{\mathbf{A}_{\alpha}\,{}^{m}e^{\varepsilon}(t)+\mathbf{B}_{\alpha}\,{}^{m}u^{\varepsilon}(t)\right\} $$

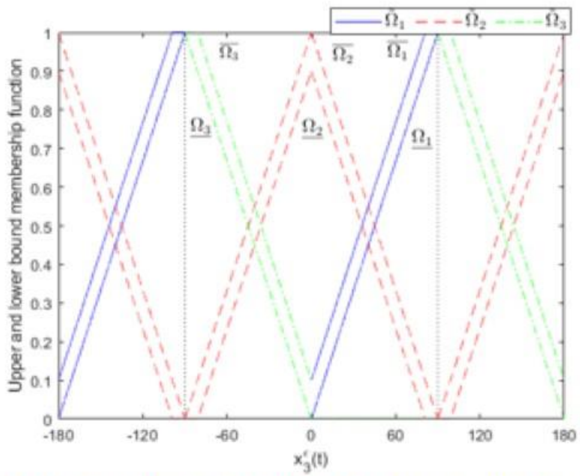

Figure 2. IT-2 membership function of nonlinear multi-ship systems

IT-2 Fuzzy Tracking Controller

$$ {}^{m}u^{\varepsilon}(t)=\sum_{\beta=1}^{2}\tilde{\Gamma}_{\beta}\left({}^{m}x_3^{\varepsilon}(t)\right)\left\{\mathbf{F}_{\beta}\,{}^{m}e^{\varepsilon}(t)\right\} $$

where $\mathbf{F}_{\beta}$ is the feedback gain

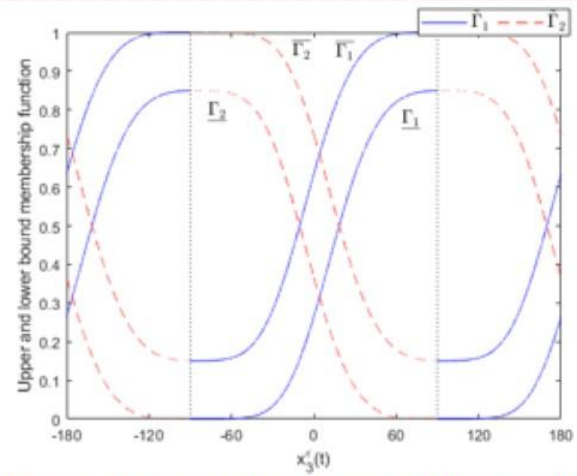

Figure 3. IT-2 membership function of fuzzy controller

IT-2 Fuzzy Controller Design

- Formation and Collision Avoidance

Stability Analysis:

Applying the fuzzy controllers to the overall fuzzy model, the following closed-loop IT-2 fuzzy models and stability criterion can be designed.

Closed-Loop IT-2 Fuzzy Model of Leader Ships

for $\varepsilon = 1,2,3,4$

$$ {}^{m}\dot{e}(t) = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\tilde{\Omega}_{\alpha}\left({}^{m}x_{3}(t)\right)\tilde{\Gamma}_{\beta}\left({}^{m}x_{3}(t)\right)\left\{\left(\mathbf{I}_{4}\otimes\left(\mathbf{A}_{\alpha}+\mathbf{B}_{\alpha}\mathbf{F}_{\beta}\right)\right){}^{m}e(t)\right\} $$

Note

If the stability of the state error dynamic ${}^{m}e^{\varepsilon}(t)$ is ensured, then the states of leader ships can achieve the desired value since

$$ \lim_{t\to\infty} {}^{m}e^{\varepsilon}(t) = {}^{m}x^{\varepsilon}(t) - {}^{m}x_{d}^{\varepsilon}(t) = 0 $$

Stability Criterion (Theorem 1)

Given the scalars $\bar{\sigma}_{\alpha\beta i_3}$ and $\underline{\sigma}_{\alpha\beta i_3}$, if there exist the positive definite matrices $\mathbf{Q}$, $\mathbf{N}_{\alpha\beta}$ and the symmetric matrix $\mathbf{M}$ such that the following sufficient conditions are all satisfied, the tracking purpose for the collision avoidance and formation can be achieved for leader ships.

(16) $\sum_{\alpha=1}^{3}\sum_{\beta=1}^{2}\left(\underline{\sigma}_{\alpha\beta i_3}\mathbf{\Phi}_{\alpha\beta} - \left(\underline{\sigma}_{\alpha\beta i_3} - \bar{\sigma}_{\alpha\beta i_3}\right)\mathbf{N}_{\alpha\beta} + \underline{\sigma}_{\alpha\beta i_3}\mathbf{M}\right) - \mathbf{M} < 0 \quad for\ i_3 = 1,2$

(17) $\mathbf{\Phi}_{\alpha\beta} - \mathbf{N}_{\alpha\beta} + \mathbf{M} < 0$

where $\mathbf{\Phi}_{\alpha\beta} = \mathbf{A}_{\alpha}\mathbf{Q} + \mathbf{B}_{\alpha}\mathbf{G}_{\beta} + \mathbf{Q}\mathbf{A}_{\alpha}^{T} + \mathbf{G}_{\beta}^{T}\mathbf{B}_{\alpha}^{T}$, $\mathbf{G}_{\beta} = \mathbf{F}_{\beta}\mathbf{Q}$, $\mathbf{Q} = \mathbf{P}^{-1}$.

IT-2 Fuzzy Controller Design

- Formation and Collision Avoidance

Formation and Collision Avoidance Control:

In this research, the tracking problem of a ship's position and yaw angle is directly considered in both formation and collision avoidance tasks. Then, the formation and collision avoidance can be completed according to the following design process.

$$^{m}u^{\varepsilon}(t)=\sum_{\beta=1}^{2}\tilde{\Gamma}_{\beta}\left(^{m}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{F}_{\beta}\,^{m}e^{\varepsilon}(t)\right\}$$

Designed $\mathbf{F}_{\beta}$

Formation Mode

$$^{f}u^{\varepsilon}(t)=\sum_{\beta=1}^{2}\tilde{\Gamma}_{\beta}\left(^{f}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{F}_{\beta}\,^{f}e^{\varepsilon}(t)\right\}$$

$$^{f}e^{\varepsilon}(t)=\left[^{f}e_{1}^{\varepsilon}(t)\quad ^{f}e_{2}^{\varepsilon}(t)\quad ^{f}e_{3}^{\varepsilon}(t)\quad ^{f}x_{4}^{\varepsilon}(t)\quad ^{f}x_{5}^{\varepsilon}(t)\quad ^{f}x_{6}^{\varepsilon}(t)\right]^{\mathrm{T}}$$

Given Individual Desired Trajectory:

$^{f}x_{1d}^{\varepsilon}(t)\quad ^{f}x_{2d}^{\varepsilon}(t)\quad ^{f}x_{3d}^{\varepsilon}(t)$ *for* $\varepsilon=1,2,3,4$

Formation Complete

Avoidance Mode

$$^{c}u^{\varepsilon}(t)=\sum_{\beta=1}^{2}\tilde{\Gamma}_{\beta}\left(^{c}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{F}_{\beta}\,^{c}e^{\varepsilon}(t)\right\}$$

$$^{c}e^{\varepsilon}(t)=\left[^{c}e_{1}^{\varepsilon}(t)\quad ^{c}e_{2}^{\varepsilon}(t)\quad ^{c}e_{3}^{\varepsilon}(t)\quad ^{c}x_{4}^{\varepsilon}(t)\quad ^{c}x_{5}^{\varepsilon}(t)\quad ^{c}x_{6}^{\varepsilon}(t)\right]^{\mathrm{T}}$$

Desired Position and Course:

$^{c}x_{1d}^{\varepsilon}(t)\quad ^{c}x_{2d}^{\varepsilon}(t)\quad ^{c}x_{3d}^{\varepsilon}(t)$

Collision Avoidance Algorithm

IT-2 Fuzzy Controller Design
- Collision Avoidance

Collision Avoidance Algorithm:

To obtain the desired position and course for keeping each ship safe from obstacles and other ships, the collision avoidance algorithm based on Artificial Potential Field (APF) is proposed as follows.

Desired Position and Course: ${}^{c}x_{1d}^{\xi}(t)$ ${}^{c}x_{2d}^{\xi}(t)$ ${}^{c}x_{3d}^{\xi}(t)$

Vortex Flow ($R_s < r \le R_v$)

X & Y Positions:
$$\begin{cases} x_{1d}(t) = x_s(t) + L\cos(\psi_d) \\ x_{2d}(t) = y_s(t) + L\sin(\psi_d) \end{cases}$$

Yaw Angle:
$$x_{3d}(t) = \psi_d = \tan^{-1}(v_{yv}/v_{xv})$$

Gradient:
$$\nabla\Theta_v = (v_{xv}, v_{yv}) = \left(-\frac{y_s(t) - y_a(t)}{{}_p r^2}, \frac{x_s(t) - x_a(t)}{{}_p r^2}\right)$$

Source Flow ($0 < r \le R_s$)

X & Y Positions:
$$\begin{cases} x_{1d}(t) = x_s(t) + L\cos(\psi_d) \\ x_{2d}(t) = y_s(t) + L\sin(\psi_d) \end{cases}$$

Yaw Angle:
$$x_{3d}(t) = \psi_d = \tan^{-1}(v_{ys}/v_{xs})$$

Gradient:
$$\nabla\Theta_s = (v_{xs}, v_{ys}) = (v_r\cos{}_p\theta + v_\theta\sin{}_p\theta, v_r\sin{}_p\theta + v_\theta\cos{}_p\theta)$$

Detected Information of Obstacles and Other Ships

Distance: ${}_p r = \sqrt{(x_s(t) - {}_p x_a(t))^2 + (y_s(t) - {}_p y_a(t))^2}$

Angle: ${}_p\theta = \tan^{-1}\left((y_s(t) - {}_p y_a(t))/(x_s(t) - {}_p x_a(t))\right)$

Simulation Results
- Multi-Ship Systems

Gains of Formation and Collision Avoidance Control:

By solving the sufficient conditions in *Theorem 1* with MATLAB, the control gains can be obtained as follows to achieve the asymptotically stability, formation and collision avoidance at the same time.

IT-2 Fuzzy Tracking Controller

$$ {}^{m}u^{\varepsilon}(t) = \sum_{\beta=1}^{2} \tilde{\Gamma}_{\beta}\left({}^{m}x_{3}^{\varepsilon}(t)\right)\left\{\mathbf{F}_{\beta}\, {}^{m}e^{\varepsilon}(t)\right\} $$

Control Gains:

$$ \mathbf{F}_1 = \begin{bmatrix} -0.6013 & -0.4007 & -0.0101 & -2.4117 & -0.2879 & -0.0029 \\ 0.7777 & -0.6088 & 0.0346 & 0.5683 & -2.6147 & 0.0690 \\ -0.0311 & 0.0050 & -0.4411 & -0.0231 & 0.0359 & -0.8179 \end{bmatrix} $$

$$ \mathbf{F}_2 = \begin{bmatrix} -0.6269 & 0.3987 & 0.0101 & -2.5153 & 0.2857 & 0.0029 \\ -0.7723 & -0.6624 & 0.0381 & -0.5650 & -2.8564 & 0.0770 \\ 0.0309 & 0.0077 & -0.4563 & 0.0229 & 0.0493 & -0.8580 \end{bmatrix} $$

when $-90^\circ \le {}^{m}x_{3}^{\varepsilon}(t) \le 90^\circ$

$$ \mathbf{F}_1 = \begin{bmatrix} 0.6013 & 0.4007 & -0.0101 & -2.4117 & -0.2879 & -0.0029 \\ -0.7777 & 0.6088 & 0.0346 & 0.5683 & -2.6147 & 0.0690 \\ 0.0311 & -0.0050 & -0.4411 & -0.0231 & 0.0359 & -0.8179 \end{bmatrix} $$

$$ \mathbf{F}_2 = \begin{bmatrix} 0.6269 & -0.3987 & 0.0101 & -2.5153 & 0.2857 & 0.0029 \\ 0.7723 & 0.6624 & 0.0381 & -0.5650 & -2.8564 & 0.0770 \\ -0.0309 & -0.0077 & -0.4563 & 0.0229 & 0.0493 & -0.8580 \end{bmatrix} $$

when $90^\circ < {}^{m}x_{3}^{\varepsilon}(t) \le 180^\circ$ and $-180^\circ \le {}^{m}x_{3}^{\varepsilon}(t) < -90^\circ$

Simulation Results
- Multi-Ship Systems

Trajectories of Multiple Ships:

Compared with the figure of control problem, the trajectories of four leader ships obtained by the IT-2 fuzzy controller design method are presented as follows.

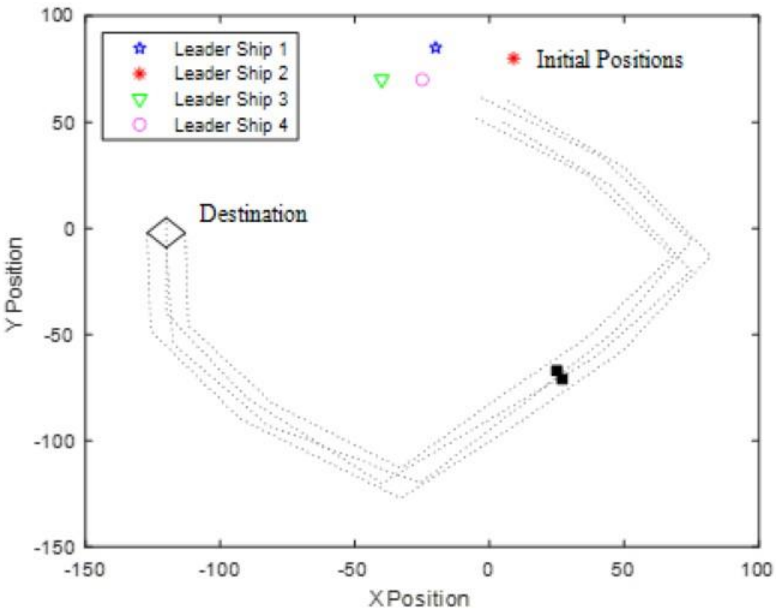

Figure 1. Formation and collision avoidance problems.

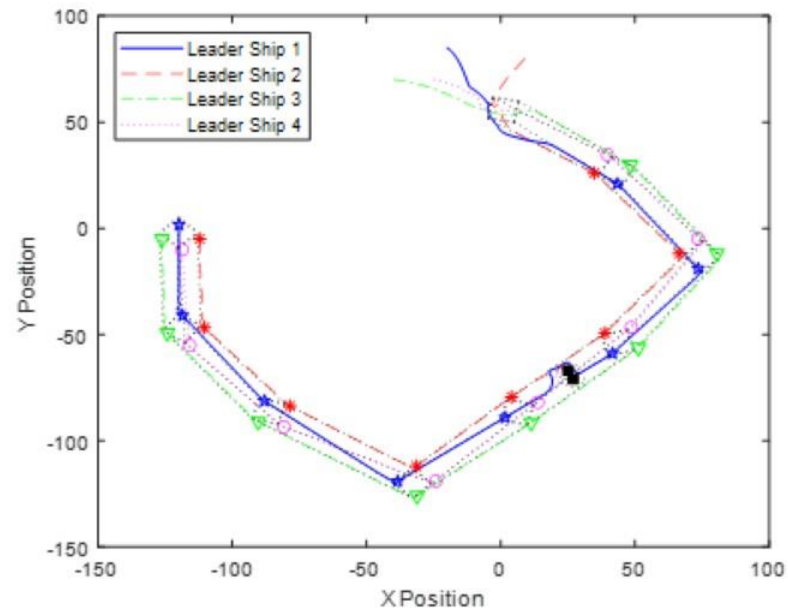

Figure 4. Trajectories of four leader ships.

Simulation Results
- Multi-Ship Systems

Trajectories of Multiple Ships:

To clearly present the collision avoidance control performance, the zoomed-in figures before completing the first formation and when avoiding two obstacles are presented as follows.

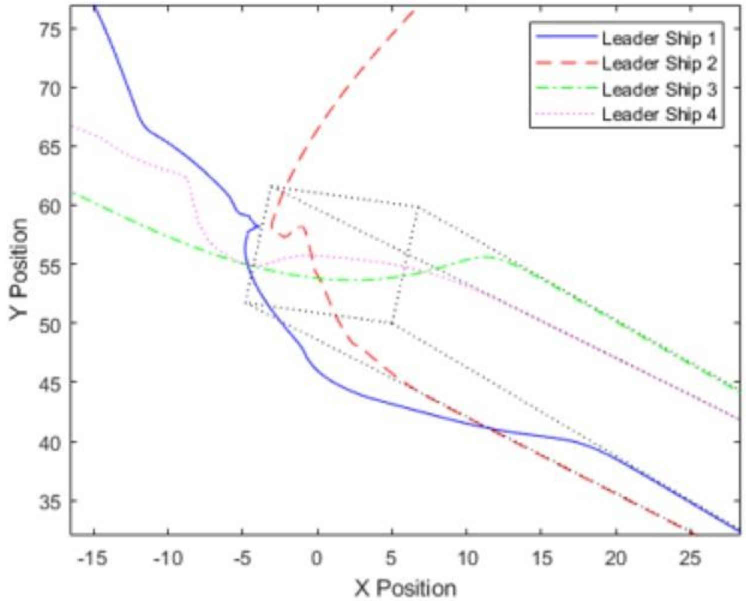

Figure 5. Trajectories of four leader ships in tracking situation.

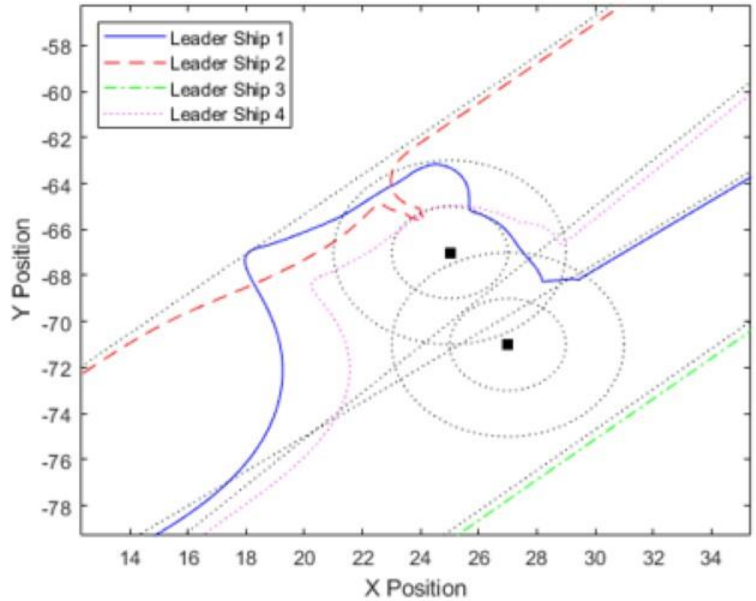

Figure 6. Trajectories of four leader ships in avoidance situation.

Conclusions & Future Works

Conclusions:

1. The fuzzy controller is designed to achieve formation and collision avoidance simultaneously by using the imperfect premise matching design method based on the IT-2 T-SFM.
2. The formation task of four leader ships is completed by their own error feedback fuzzy tracking controller such that the communication problem can be efficiently avoided. In addition , the tracking controller can be extended to the application of collision avoidance.
3. Both the vortex flow and source flow are considered to develop the avoidance algorithm based on the APF concept.

Future Works:

1. To make the application of the IT-2 fuzzy controller more comprehensive for multi-ship systems, the containment problem should also be considered.
2. Enhancing the reliability of the IT-2 fuzzy controller, the other performance requirements such as disturbance and delay can be considered.

Thank you for your attention!